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Mathematical modelling of thin-layer drying of carrot

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Abstract. This paper presents a new mathematical model for thin-layer drying process, which was verified with experimental data. A laboratory scale static-tray dryer was used to obtain the experimental data. Carrot cubes were used for drying experiments. Experiments were performed at air temperatures of 50, 60 and 70°C at constant air velocity of 1 m s^{-1}. Fifteen different mathematical drying models and the new model were compared based on the correlation coefficient, root mean error and reduced chi-square between the experimental and predicted moisture ratios. Consequently, of all the drying models, the proposed model was selected as the best mathematical model for describing the drying kinetics of carrot cubes. Constants related to the new model are reported and correlated with air temperature using Arrhenius equation.

Keywords: carrot, thin layer drying, mathematical modelling, Arrhenius equation

INTRODUCTION

Carrot (*Daucus carota* L.) is the most commonly used vegetable for human nutrition (Doymaz, 2004; Erenturk and Erenturk, 2007). Carrots are an excellent source of Beta Carotenoids reported to prevent cancer, vitamin A and potassium, and contain cholesterol lowering pectin, vitamin C, vitamin B6, thiamine, folic acid, and magnesium. Dried carrots are used in dehydrated soups and in the form of powder in pastries and sauces. Drying of food, fruit, meat and other agricultural products dates back to before the discovery of fire. Conventional air-drying is the most frequently used dehydration operation in food industry (Górnicki and Kaleta, 2007). It also brings about substantial reduction in weight and volume, and minimises packaging, storage and transportation costs. One of the most important aspects of drying technology is the modelling of the drying process (Khazaei and Daneshmandi, 2007). The prediction of drying kinetics of agricultural products under various conditions is very useful in the design and optimisation of dryers.

Many models have been proposed by several authors for the estimation of drying rates of biological materials, that have finally led to different expressions for the prediction of drying rates (Table 1), yet none of the empirical and semi-empirical models can be used over a wide range of foods and drying conditions. Each of the empirical and semi-empirical models may account for varying degrees of effect of drying conditions on drying process which must be considered during the drying process. Since the drying kinetics is greatly affected by air temperature, air velocity, material size, drying time (Erenturk and Erenturk, 2007; Khazaei *et al.*, 2008), the new model could be used for the modelling of drying kinetics as its constants (k_1 and k_2) include the effect of drying variables on drying kinetics.

The aim of this research was to develop a new mathematical model for predicting the drying kinetics of biological products.

MATERIALS AND METHODS

Carrots procured from a local market were used in the study. Samples were stored in a refrigerator at 5°C prior to the drying experiments. The initial moisture content of the carrot samples was determined by the oven drying method. About 50 g of sample was dried in an oven at 105±2°C for 4 h. The experiments were replicated three times.

Detailed information on the experimental system and its design was presented in the previous study (Aghbashlo *et al.*, 2008). The dryer consists of an adjustable centrifugal blower, air heating chamber (4.5 kW), drying chamber, system controller, inverter (Lenze 8300, Gmbh and co KG Aerzen, Postfach, Germany) and sample tray. The experiments were preformed at air temperatures of 50, 60 and 70°C at constant air velocity of 1 m s^{-1}. The dryer was installed in an environment with relative humidity of about 35-42% and ambient

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temperature of about 18-23°C. To reduce the undesirable effects of temperature and humidity of air on drying experiment, the drying chambers and tunnel length were insulated with rock wool and wood. The drying tray was isolated using the glass cylinder and hot air exhaust from the upper part of the glass cylinder. The dryer had an automatic temperature controller with an accuracy of ±0.1°C. Air velocity was kept at the mentioned value of 1 m s^{-1} with accuracy of ±0.05 m s^{-1} using PROVA AVM-07 (TES, Co, Taipei, Taiwan) anemometer. The air velocity was fixed by means of an inverter which directly acted on the blower motor (1.5 kW). Hot air orientation on the samples was vertical. At the start of each experiment, the carrots were washed and cut into cubes having the dimensions of 2×2×2 cm, using a mechanical cutter Samples used in experiment were about 150 g in weight. The dried samples were weighed using a precision balance with an accuracy of 0.01 g. Weighing was made every 20 min. During the drying process, experiments continued until the mass change between two weighings was less than 0.05 g. Drying experiments were repeated twice. Temperature controller fixed the temperature of the air chamber with ±1°C. Before the start of any experiments, the dryer system was started in order to achieve the desired steady state condition.

Moisture ratio (*MR*) of carrot cubes during the thin layer drying experiments was calculated using Eq. (1):

$$MR = \frac{M_t - M_e}{M_0 - M_e}, \tag{1}$$

where: M_t is the current moisture content (% d.b.), M_e is equilibrium moisture content (% d.b.) and M_o is initial moisture content (% d.b.). Equilibrium moisture content is defined as the moisture content when vapour pressure of water present in the food material has reached equilibrium with its surroundings (drying air).

Many models have been used by several researchers for the estimation of moisture ratio of materials during drying process. These are shown in Table 1. Moreover, in this paper a new model is used for modelling of drying kinetics of biological material:

$$MR = \exp\left(-\frac{k_1 t}{1 + k_2 t}\right), \tag{2}$$

where: k_1 and k_2 are the constants (min^{-1}) and t is time (min).

All pervious models and the new model were fitted to experimental data by using non-linear least squares regression solved by the Levenberg-Marguardt numerical method. Three criteria were used to evaluate the models – coefficient of determination (R^2), reduced chi-square (χ^2) and root mean square error (*RMSE*).

RESULTS AND DISCUSSION

The initial moisture content of carrot was observed to be 8.3±0.36 (% d.b.). The equilibrium moisture content of carrot slices was obtained using the dynamic method at different air temperatures used in the drying experiments (Kashaninejad *et al.*, 2007). The equilibrium moisture content of carrot slices varied from 0.133±0.019 to 0.284±0.031 (% d.b.). The data obtained from the experiment were converted to dimensionless moisture ratio by using Eq. (1).

Figure 1 exhibits the variation of moisture ratio as a function of time. The moisture ratio of the samples decreased continually with drying time. As expected, increase in the temperature of drying air reduces the time required to reach any given level of moisture ratio since the heat transfer increases. This can be explained by increasing temperature difference between the drying air and the product and the resultant water migration.

Figure 2 exhibits the variation of drying rate as a function time. As expected, increase in the drying air temperature increases the drying rate because higher air temperature causes a higher reduction of moisture content – in other words, at high temperatures the transfer of heat and mass is high and water loss is excessive. As can be seen from Fig. 2, no constant rate period was observed in the drying of carrot cubes; the drying process took place at a falling rate period.

As seen from Figs 1 and 2, one of the main factors influencing the drying kinetics of carrot is the air temperature. Drying time was decreased depending on the increase in the air temperature. Drying down to 0.1 moisture ratios in the dryer took about 780, 500 and 280 min at the air temperatures of 50, 60 and 70°C, respectively.

All pervious models and the new model were fitted to experimental data by using *MATLAB* computer program. The statistical results from fifteen previous models and the new drying model, such as determination coefficient, chi-square and root mean square error, are given in Table 2. The results indicate that the highest values of R^2 and the lowest χ^2 values and *RMSE* could be obtained by the new drying model. Generally, the R^2, χ^2 and *RMSE* values of the new model ranged from 0.9996 to 0.9999, 2.032×10^{-8} to 1.256×10^{-6} and 0.002491 to 0.004358, respectively. Demir *et al.* (2007), Midilli *et al.* (2002) and diffusion approach models give the closest result to the present model, but the new model is simpler than those models.

The constants related to the new model are reported in Table 3. The values of k_1 and k_2 varied between 0.005339 to 0.009175 min^{-1} and 0.000435 to 0.001032 min^{-1}, respectively.

The dependence of the drying constants on air temperature can be described by the Arrhenius model:

$$k_1 = 54.18\exp\left(\frac{2991.2}{T_{abs}}\right) \qquad R^2=0.9586, \tag{3}$$

$$k_2 = -4.27\,10^{-10}\exp\left(\frac{-4766.7}{T_{abs}}\right) \quad R^2=0.9544, \tag{4}$$

where: T_{abs} is the temperature in absolute scale (K).

Table 1. Drying models

Number	Model	Model equation	References
1	Lewis	$MR = \exp(-kt)$	Lewis (1921)
2	Page	$MR = \exp(-kt^n)$	Page (1949)
3	Modified Page	$MR = \exp[(-kt^n)]$	Overhults *et al.* (1973)
4	Henderson and Pabis	$MR = a\exp(-kt)$	Westerman (1973)
5	Logarithmic	$MR = a\exp(-kt) + c$	Togrul and Pehlivan (2003)
6	Two-term	$MR = a\exp(-k_0 t) + b\exp(k_1 t)$	Henderson (1974)
7	Two-term exponential	$MR = a\exp(-kt) + (1-a)\exp(-kat)$	Sharaf-Elden *et al.* (1980)
8	Wang and Singh	$MR = 1 + at + bt^2$	Wang and Singh (1978)
9	Diffusion approach	$MR = a\exp(-kt) + (1-a)\exp(-kbt)$	Demir *et al.* (2007)
10	Verma *et al.*	$MR = a\exp(-kx) + (1-a)\exp(-gx)$	Verma *et al.* (1985)
11	Modified Henderson and Pabis	$MR = a\exp(-kt) + b\exp(-gt) + c\exp(-ht)$	Karathanos (1999)
12	Simplified Fick's diffusion equation	$MR = a\exp(-k(t/L^2))$	Diamente and Munro (1991)
13	Midilli *et al.*	$MR = a\exp(-kt^n) + bt$	Midilli *et al.* (2002)
14	Demir *et al.*	$MR = a\exp(-kt)^n + b$	Demir *et al.* (2007)
15	Weibull	$MR = \exp\left(-\left(\frac{t}{a}\right)^b\right)$	Corzo *et al.* (2008)

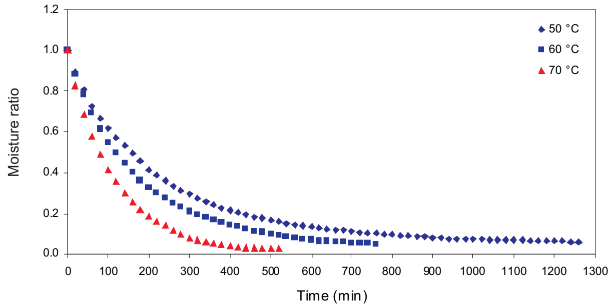

Fig. 1. Variation of moisture ratio with drying time.

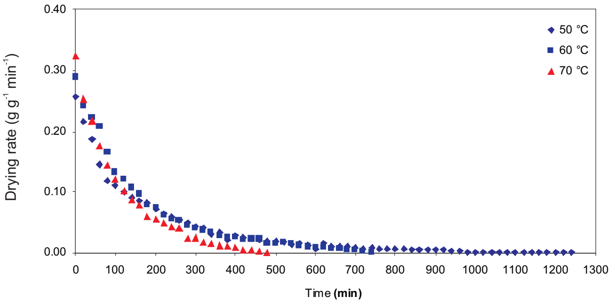

Fig. 2. Variation of drying rate with drying time.

The high value of coefficient of determination obtained for Eqs (3) and (4) is additional evidence for the suitability of the new model to predict drying behaviour of biological material. The generalised drying models are valid in the air temperature range of $50 \leq T \leq 70$°C and 1.0 m s^{-1} drying air velocity.

Figure 3 shows the plot of the experimental data of moisture ratio versus the predicted values of moisture ratio calculated by the new model (using Eqs (3) and (4)). The data points are banded around a 45° straight line, demonstrating the suitability of the new model for describing the thin-layer drying behaviour of carrot.

CONCLUSIONS

1. The drying model proposed in this study could be an effective tool for the modelling of drying behaviour of biological material.

2. Among 15 pervious models, the new model gave the best results and showed good agreement with the experimental data obtained from the experiments including the thin layer drying process.

3. The new model constants of k_1 and k_2 can be predicted as a function of air temperature in the form of the Arrhenius equation.

Table 2. Statistical results of the drying models

Model	50°C			60°C			70°C		
	R^2	χ^2	*RMSE*	R^2	χ^2	*RMSE*	R^2	χ^2	*RMSE*
1	0.9609	1.485E-03	0.044161	0.9889	6.700E-04	0.026270	0.9985	7.193E-06	0.010253
2	0.9940	3.365E-04	0.017339	0.9985	1.190E-05	0.009788	0.9998	8.361E-08	0.003538
3	0.9738	6.564E-03	0.036439	0.9985	1.190E-05	0.009788	0.9997	2.760E-07	0.004769
4	0.9738	6.566E-04	0.036439	0.9921	3.280E-04	0.022418	0.9990	3.227E-06	0.008643
5	0.9984	2.313E-05	0.008987	0.9989	5.740E-06	0.008324	0.9997	2.650E-07	0.004769
5	0.9995	1.696E-06	0.004734	0.9939	1.870E-04	0.020307	0.9994	8.724E-07	0.006633
7	0.9871	1.588E-03	0.025557	0.9992	3.030E-06	0.006948	0.9998	1.095E-07	0.003710
8	0.8127	3.359E-01	0.097459	0.9188	3.541E-02	0.072261	0.9319	1.500E-02	0.071353
9	0.9993	4.327E-06	0.005910	0.9994	1.550E-06	0.006174	0.9998	8.026E-08	0.003538
10	0.9993	4.327E-06	0.005910	0.9996	6.770E-07	0.004946	0.9998	8.026E-08	0.003538
11	0.9993	4.115E-06	0.005910	0.9994	1.420E-06	0.006174	0.9997	1.123E-07	0.004254
12	0.9738	6.564E-03	0.036439	0.9889	6.520E-04	0.026270	0.9985	6.916E-06	0.010253
13	0.9994	2.420E-06	0.005091	0.9996	6.580E-07	0.004946	0.9998	1.008E-07	0.003710
14	0.9984	2.275E-05	0.008987	0.9994	1.510E-06	0.006174	0.9997	1.230E-07	0.004254
15	0.9940	3.365E-04	0.017339	0.9985	1.190E-05	0.009788	0.9998	8.361E-08	0.003538
New model	0.9996	1.256E-06	0.004358	0.9999	8.330E-08	0.002491	0.9999	2.032E-08	0.002538

Table 3. Constants related to the new model

Temperature (°C)	k_1	k_2
50	0.005339	0.001032
60	0.006403	0.000779
70	0.009175	0.000435

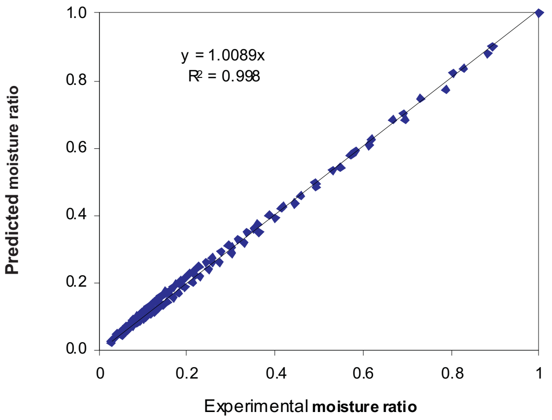

Fig. 3. Experimental moisture ratio versus predicted moisture ratio values by the new model.

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