Abstract. This paper deals with the correlation between global solar radiation and sunshine duration for the south-eastern part of Romania. Daily data on sunshine duration ($n$) and global solar radiation ($Rs$) from the Constanta weather station were used for a period of 30 years, between 1971 and 2000. Coefficients $a$ and $b$ from Angström's equation calculated for the Constanța location were very close to those recommended worldwide. About 24% of extra-terrestrial radiation penetrates the atmosphere on a fully-clouded ($n = 0$) day, and about 74-75% on a clear-sky day, respectively. The coefficients computed from the mean monthly data did not practically differ from those computed using the mean daily data. Both data pairs can be used in estimating global solar radiation for the Black Sea coastal area of Romania. The direct correlation equations between $Rs$ and $n$ can be also used in estimating global solar radiation. Similarly, the regression equation between $Rs$ and temperature could be utilized too. However, they do not present the same consistency or physical significance as Angström's equation type.

Keywords: extra-terrestrial radiation, air temperature, relative air humidity

INTRODUCTION

Global solar radiation ($Rs$) has a fundamental importance for life on earth. $Rs$ is used in various types of calculation, e.g., in estimating reference evaporation. However, its measurement is costly and time-consuming. Efforts have therefore been made in order to find a way to estimate $Rs$. Thus, Kimball [6] found a correlation that had an approximately linear shape between $Rs$ and sunshine duration ($n$), and Angström [2] was the first scientist known to mathematically relate the two solar properties as a fraction of the global solar radiation of a clear-sky day. Later on Prescott [8] utilized the following formula to estimate solar radiation:

$$Rs = [a + b (n/N)] Ra,$$

where $Rs$ - the global solar radiation (expressed in this paper in MJ m$^{-2}$ day$^{-1}$), $n$ - the real sunshine duration (h), $N$ - the day’s length (h), $Ra$ - the extraterrestrial radiation (MJ m$^{-2}$ day$^{-1}$), $a$ - the intersection of the line (constant) expressing the fraction of $Ra$ reaching the earth on cloud-covered days when $n = 0$, $b$ - the coefficient of regression, and $a + b$ - a fraction of $Ra$ reaching the earth on clear-sky days, when $n = N$.

At sea level for clear-sky periods, Angström’s formula describing $Rs$ ($Rso$) becomes:

$$Rso = (a + b) Ra,$$

where the other symbols remained unchanged.

Studies on the $a$ and $b$ coefficients of Angström’s formula have previously been published by various authors [3,4,7].

Based on many measurements made at various locations on the Earth and published by more than one author, Allen et al. [1] recommended the values of $a = 0.25$ and $b = 0.50$ in estimating $Rs$, when there is available data on sunshine duration and direct measurements on $Rs$ are missing. However, these values have a relatively high degree of variation on the Earth and their more precise knowledge in zones of geographic, agricultural, or hydrological interest is the permanent purpose of scientists who could lead to an improvement in global solar radiation evaluation and its derivative indicators.

The purpose of the present paper is to determine through statistical analysis the $a$ and $b$ coefficients from Angström’s formula that can be used later in estimating global solar radiation for the south-eastern part of Romania.

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MATERIALS AND METHODS

Daily data on sunshine duration (\(n\)) and global solar radiation (\(Rs\)) from the Constanta weather station located in the south-eastern part of Romania were used for a period of 30 years, between 1971 and 2000. The daily data on \(N\) and \(Ra\) were computed with the relationships given by Raes et al. [9]. In order to determine the coefficients of the general regression equation more than one step were followed. First of all the series of the monthly ratios between the \(Rs/Ra\) and \(n/N\) values were computed, and then the regression equations between them. In the same time, the \(Rs\) and \(n\) parameters were also directly correlated. Then, the regression equations between the daily values of the \(Rs/Ra\) and \(n/N\) ratios were computed too. Coefficients of correlation or determination were then tested for their significance using the Fisher test. In the figures that follow one star means significant, two stars mean distinctly significant and three stars show a highly significant correlation.

RESULTS

Correlations found using mean monthly values

The correlation between \(Rs/Ra\) and \(n/N\) as well as the correlation between \(Rs\) and \(n\), respectively, were shown in Figs 1 and 2. The regression equations found through calculation were:

\[
Rs = 2.3108n - 0.2734
\]

\[
R = 0.933^{***}
\]

\[
Rs/Ra = 0.237 + 0.511(n/N)
\]

\[
R = 0.887^{***}
\]

\[
Rs/Ra = -0.2996(n/N)^2 + 0.7969(n/N) + 0.178
\]

\[
R = 0.891^{***}
\]

Fig. 1. Correlation between \(Rs/Ra\) and \(n/N\) for the 1971-2000 period in Constanta, Romania, mean monthly values.

Fig. 2. Correlation between global solar radiation (\(Rs\)) and sunshine duration (\(n\)) in Constanta, Romania, period 1971-2000, mean monthly values.
\[ Rs = (0.237 + 0.511 \frac{n}{N}) Ra \quad (3) \]
\[ Rs = 2.3108 n - 0.2734, \quad (4) \]

with the coefficients of determination \( R^2 \) of 0.7865*** for Eq. (1) and 0.8705*** for Eq. (2), both highly significant. In the Eq. (1) \( Rs \) and \( Ra \) were expressed in the same measure units (MJ m\(^{-2}\) day\(^{-1}\)), in the Eq. (2) \( Rs \) was expressed in MJ m\(^{-2}\) day\(^{-1}\) and \( n \) in h day\(^{-1}\).

So, the \( a \) constant attained 0.237 in this case, whereas the \( b \) constant was as high as 0.511, but the coefficients of determination showed that, in the first case about 21% from the \( Rs \) variation was attributed to causes other than the variation of the \( n/N \) ratio, and in the second case about 13% from the \( Rs \) variation remained outside the \( n \) parameter.

The regression equation for clear sky days is:

\[ R_{so} = 0.748 Ra. \quad (5) \]

In order to increase confidence in the correlations between \( Rs/Ra \) and \( n/N \) a degree-2 polynomial function was also used, but this attempt improved the determination coefficient by only 1%. This did not justify the change of the equation type from linear, with a known physical meaning, to polynomial. Sandoval and Shaw [10] earlier reported a curved-linear correlation between the parameters discussed, but also in this case this curved-linear regression equation only brought a slight improvement in the correlation coefficient versus the linear regression equation. Similarly, Harris [5] tested more regression equation types, but neither of them (curved-linear) was clearly better than the linear.

### Correlations found using mean daily values

Regression equations using mean daily values of the same ratios studied or the climate elements above mentioned were next computed. These are:

\[ Rs = (0.243 + 0.50 \frac{n}{N}) Ra \quad (6) \]
\[ Rs = 1.64 n + 3.89, \quad (7) \]

with \( R^2 = 0.78*** \) for Eq. 6 and 0.748*** for Eq. (7), to which the total number of data pairs studied was almost 11000 (Figs 3 and 4).

### The multiple correlation between the mean monthly values of Rs on the one hand and sunshine duration, air temperature, and relative air humidity on the other hand

Figure 5 shows the correlation between \( Rs \) and air temperature (\( Tm \)). It is linear in shape and is direct and highly significant, with \( R^2 = 0.74*** \). This regression equation can also be used in estimating \( Rs \) as it has a relatively high confidence degree.
Fig. 4. Correlation between global solar radiation ($Rs$) and sunshine duration ($n$) in Constanta, Romania, the 1971-2000 period, mean daily values (about 11000 of data pairs).

$$Rs = 1.6378 \, n + 3.8897 \quad R = 0.865^{***}$$

$$Rs = 0.0518 \, n^2 + 1.0009 \, n + 4.7716 \quad R = 0.871^{***}$$

Fig. 5. Correlation between $Rs$ and $Tm$ in Constanta, Romania, the 1971-2000 period, mean monthly values.

$$Rs = 0.8052 \, Tm + 4.6656 \quad R = 0.860^{***}$$
Figure 6 shows the linear, inverse and highly significant correlation between $R_s$ and relative air humidity ($RH$), where $R^2=0.372^{***}$, but this cannot be used in estimating $R_s$ due to the low value of squared $R$.

These types of single correlations suggested the idea of computing the parameters of a multiple correlation between $R_s$ on the one hand and $n$, $Tm$ and $RH$ on the other hand. These parameters can be viewed in Table 1.

As can be seen, in the case of the Angström type equation, $R^2$ remained practically the same for the mean daily values as it was for the mean monthly values, namely 0.78, while the $a$ and $b$ constants studied suffered very small changes. Not the same thing can be stated on the direct correlation between $R_s$ and $n$, where $R^2$ decreased for the mean daily values at 0.748***, remaining also highly significant. The decrease probably occurred due to the higher dispersion of the mean daily values of $R_s$ and $n$.

Analyzing the coefficients of determination, $R^2$, it can be stated, for instance, that the strength between $R_s$ and $Tm$ decreased from the simple (total) correlation ($R^2=0.74^{***}$) to the first order, partial correlation when $RH$ ($R^2=0.624^{***}$) or $n$ ($R^2=0.001$) were kept constant, or to the second order, partial correlation, when both $RH$ and $n$ were kept constant at the same time ($R^2=0.0007$). This can be explained through the correlation between the investigated variables: $Tm$, $RH$, and $n$, which were initially assumed independent as a hypothesis of work and which were, in turn, correlated between them. For example, it was well known that $Tm$ and $RH$ were generally correlated. So, $Tm$ was found to be correlated with $RH$ ($R^2=0.314^{***}$) and with $n$ ($R^2=0.843^{***}$), while $RH$ was correlated with $n$ ($R^2=0.445^{***}$).

The second correlation, between $R_s$ and $RH$, had a weak intensity ($R^2=0.372^{***}$) and was not worth detailed analysis, but the correlation between $R_s$ and $n$ showed the maximum strength. $R^2$ also decreased in this case, from the simple correlation ($R^2=0.870^{***}$) to the partial ones when $Tm$ was kept constant ($R^2=0.493^{***}$), or when both $RH$ and $Tm$ were kept constant simultaneously ($R^2=0.443^{***}$), respectively.

Finally, within the multiple-type correlation a determination coefficient was yielded as high as that from the simple-type correlation between $R_s$ and $n$ ($R^2=0.870^{***}$), showing that no progress was obtained in estimating $R_s$ when, in addition to $n$, $Tm$ and $RH$ were also considered in calculation.

**DISCUSSION**

Doorenbos and Pruitt [4] presented monthly values for the $a$, $b$ and $a+b$ coefficients of Angström’s equation as obtained by many authors worldwide. These values, spatial averages for the 54° northern latitude were: 0.21, 0.55 and 0.76, respectively, while for the 36° northern and southern latitude they were: 0.23, 0.53 and 0.76, respectively. For the two hemispheres at low, north and south latitudes, the above-mentioned coefficients were 0.28, 0.49 and 0.77, respectively, for 24° latitude, 0.26, 0.50 and 0.76, respectively, for 13° latitude, and 0.25, 0.49 and 0.74 for 3° latitude.

The individual values of the coefficients discussed varied from place to place, e.g., from $a = 0.10$ for Senegal.

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**Fig. 6.** Correlation between $R_s$ and $RH$ in Constanța, Romania, the 1971-2000 period, mean monthly values.
and $a = 0.15$ for Belgium, to $a = 0.33$ for South Africa. The $b$ coefficient also presented a wide range, from 0.38 in Nigeria to 0.62 in Canada.

The $a$, $b$ and $a+b$ coefficients found in the present paper: $0.24$, $0.51$ and 0.75, respectively, obtained from monthly values, or 0.24, 0.50 and 0.74, respectively, obtained from daily values, were very close to those reported by Doorenbos and Pruitt [4] for the 54 and 36° north and south latitudes. At the same time these values were very close to those recommended by Allen et al. [1], namely $a = 0.25$, $b = 0.50$ and $a + b = 0.75$.

Thus, the present paper particularized these coefficients for the south-eastern zone of Romania and could introduce its contribution into the estimation of global solar radiation, using the correlations obtained here and the sunshine duration data.

CONCLUSIONS

1. Coefficients $a$ and $b$ from Angström’s equation calculated for the Constanţa location are very close to those recommended worldwide. About 24% of extraterrestrial radiation penetrates through the atmosphere on a fully-clouded ($n = 0$) day, and about 74-75% on a clear-sky day, respectively.

2. The coefficients computed from the mean monthly data did not differ practically from those computed from mean daily data. Both data pairs can be used in estimating global solar radiation for the Black Sea coastal area of Romania.

3. The direct correlation equations between $Rs$ and $n$ can be also used in estimating global solar radiation. Similarly, the regression equation between $Rs$ and temperature could be utilized too. However, they do not present the same consistency and physical significance as the Angström equation type.

REFERENCES


