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MULTIVARIATE ANALYSIS OF HIERARCHICAL CLASSIFICATION IN SPLIT PLOT DESIGN WITH RESPECT TO THE CONTROL

I. Kuna-Broniowska

University of Agriculture, Akademicka 13, 20-934 Lublin, Poland
E-mail: izakuna@ursus.ar.lublin.pl

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A b s t r a c t. The paper describes multivariate analysis of variance in the split plot design when the so-called control treatment B^0 is allocated to an additional subplot. The factor B^1 allocated to small plots is subject to the two-stage classification, where the levels of the second stage dependent on the levels of the first stage. This dependence is treated as a hierarchical classification of two factors: B^1- the first stage levels and C (inside B^1) - the second stage levels. A linear model of the experiment consisting of two forms is used, one form for the plots where all the factors are present and the other for control plots. Beside that, for the sake of better visualisation of the experiment, the component models are based on two different vectors of the grand means. Tests of hypotheses for the comparison with the control are presented. The discussed theory is illustrated by an agrophysical experiment conducted according to such a design.

K e y w o r d s: contrasts, control object, hierarchical classification, multivariate analysis of variance, split plot design

Classification AMS 1993: 62H12, 62H15.

INTRODUCTION

Split plot designs are widely used in field experiments. The problem of the comparison of group objects with control objects often occurs in variety, fertilization and feeding experiments. Recently, agrophysical experiments based on split-plot design have been carried out. These experiments need new elaboration of statistical analysis with respect to the control object and few-stage classification of one factor. The literature on the subject of block designs for univariate analysis of variance is extensive. The problem of one or more control objects has been considered with respect to development of experimental design [2,5,11, 13] and with respect to statistical analysis of such designs [1,3,9]. Beside that, this problem was considered in the light of the assumption of the equivalence in alternative hypothesis for the contrasts between test objects and the control object [6]. Another important problem, is determination of sample sizes for the comparison of k treatments against a control [7,8,12].

In this paper a multivariate analysis of variance, for a new model of experiment with two factors A and B^1 and with an additional control object B^0 set up according to the split plot design is considered [10]. An experiment, in which the levels of the A factor (A_j , j=1,2, ...,a) were randomly allocated to a whole-plots. Inside each level A_j of the factor A, bc levels of the second B^1 factor and one control B^0 object were allocated to bc +1 plots. The bc levels of B^1 are obtained, as a result of a two stage classification, in the following way: there are b first stage levels B_k (k=1,2,...,b) of B^1 and c second stage levels $C_l(B_k)$ (l=1,2,...,c) inside each level B_k of B^1. These c levels will be treated as the levels of the third factor C inside B^1. Together, the factor B^1 and the control object B^0 are denoted as B. A hierarchical classification was adopted because of the character of the $C_{l(}B_{k)}$ levels of the C factor that dependent on the B_k levels of the B^1 factor. Additional fixed effects of C and appropriate interaction effects were extended a classical linear model for the split plot design. The effects of blocks are treated as fixed effects. Random errors are the same as in the classical model, since the experiment was arranged according to the scheme of the split plot design with two factors. The goal of this paper is to present the multivariate analysis of variance for a new form of the model of such an experiment given by Kuna-Broniowska and Przybysz [10]. The matrices with appropriate sums of products for a particular null hypotheses are based on the quadratic forms obtained in univariate analysis of variance [10]. The discussed theory is widely illustrated by an example of an experiment conducted according to such a design. The example is based on the data of the agrophysical experiment conducted in University of Agriculture in Lublin, where a factor was subjected to two-stage classification with a relation between the first and second levels.

MODEL

The observations for the h-th variable of the experiment conducted in r blocks, can be described according to one of the two linear models:

$$y_{ij00h} = \mu_{0h} + \rho_{ih} + \alpha_{jh} + e_{ijh} + \beta_{0h} + (\alpha\beta)_{j0h} + e_{ij00h} \qquad (1)$$

for the control plots, where the B^1 and C factors are not present,

$$y_{ijl(k)h} = \mu_{1h} + \rho_{ih} + \alpha_{jh} + e_{ijh} + \beta_{kh} + \gamma_{l(k)h} + (\alpha\beta)_{jkh} + (\alpha\gamma)_{jl(k)h} + e_{ijl(k)h} \qquad (2)$$

for the other plots.

The joined model of the complete experiment takes the following form:

$$y_{ijl(k)h} = \begin{cases} \mu_{1h} + \rho_{ih} + \alpha_{jh} + e_{ijh} + \beta_{kh} \\ +\gamma_{l(k)h} + (\alpha\beta)_{jkh} + (\alpha\gamma)_{jl(k)h} \\ +e_{ijl(k)h} \quad \text{for} \quad k \neq 0 \wedge 1 \neq 0 \\ \mu_{0h} + \rho_{ih} + \alpha_{jh} + e_{ijh} + \beta_{0h} \\ +(\alpha\beta)_{j0h} + e_{ij00h} \ \text{for}\ k = 0 \wedge 1 = 0 \end{cases} \qquad (3)$$

where: $h = 1,..., p$; $i = 1,...,r$; $j = 1,...,a$; $k = 0,...,b$; $l = 0,...,c$; μ_{1h} and μ_{0h} are general means, ρ_{ih} is the effect of the i-th block, α_{jh} is the effect of the j-th level of A, β_{kh} is the effect of the k-th level of B^1, $\gamma_{l(k)h}$ is the effect of the l-th level of the C factor inside the k-th level of B^1, $(\alpha\beta)_{jkh}$ and $(\alpha\gamma)_{jl(k)h}$ are relevant interaction effects, e_{ijl} and $e_{ijl(k)h}$ are experimental errors.

In the model (3) there are two groups of general means, namely: μ_{1h} and μ_{0h}. This point of view was adopted because of future estimators and analyses.

Using the matrix notation and assuming a traditional data arrangement, namely according to the blocks, next the A, B factors and finally a C model (3) can be written:

$$\mathbf{Y'}_{N\times p} = \mathbf{X}_M \mu_{2\times p} + \mathbf{X}_R \rho_{r\times p} + \boldsymbol{X}_A \alpha_{a\times p} + \boldsymbol{X}_B \beta_{(b+1)\times p} + \boldsymbol{X}_{C(B^1)} \gamma_{bc\times p} + \boldsymbol{X}_{AB} \alpha\beta_{a(b+1)\times p} + \boldsymbol{X}_{AC(B^1)} \alpha\gamma_{abc\times p} + \boldsymbol{I}_N \mathbf{e'}_{2N\times p} \quad (4)$$

where: $N = ra\,(bc+1)$, $\mathbf{Y'}$ is the $(N \times p)$ matrix of observations,

$\mathbf{X}_M = \mathbf{1}_{ra\times 1} \otimes \begin{bmatrix} \mathbf{1}_{bc\times 1} & \mathbf{0}_{bc\times 1} \\ 0 & 1 \end{bmatrix}$ is the design matrix for general means occurring by $\mu = \begin{bmatrix} \mu_{11\times p} \\ \mu_{01\times p} \end{bmatrix}$,

$\mathbf{X}_R = \mathbf{I}_r \otimes \mathbf{1}_{a(bc+1)\times 1}$ is the design matrix for the R blocks, ρ is the $(r \times p)$ matrix of fixed effects of blocs,

$\mathbf{X}_A = \mathbf{1}_{r\times 1} \otimes \mathbf{I}_a \otimes \mathbf{1}_{bc+1}$ is the design matrix for the A factor,
α is the $(a \times p)$ matrix of fixed effects of A factor,
$\boldsymbol{X}_B = \mathbf{1}_{ra\times 1} \otimes \begin{bmatrix} \mathbf{I}_b \otimes \mathbf{1}_{c\times 1} & \mathbf{0}_{bc\times 1} \\ \mathbf{0}_{1\times b} & 1 \end{bmatrix}$ is the design matrix for the B factor,
$\beta_{(b+1)\times p} = \left[\beta_{1p\times 1},\ldots, \beta_{bp\times 1}, \beta_{0p\times 1}\right]'$ is the $((b+1) \times p)$ matrix of fixed effects of the B factor, $\mathbf{X}_{C(B)} = \mathbf{1}_{ra\times 1} \otimes \begin{bmatrix} \mathbf{I}_{bc} \\ \mathbf{0}_{1\times bc} \end{bmatrix}$ is the design matrix for the C(B^1) factor, γ is the $(bc{\times}p)$ matrix of the fixed effects of the C factor,
$\mathbf{X}_{AB} = \mathbf{1}_{r\times 1} \otimes \mathbf{I}_a \otimes \begin{bmatrix} \mathbf{I}_b \otimes \mathbf{1}_{c\times 1} & \mathbf{0}_{bc\times 1} \\ \mathbf{0}_{1\times b} & 1 \end{bmatrix}$ is the design matrix for the AB interaction,
$\alpha\beta = \left[\alpha\beta_{11p\times 1},\ldots,\alpha\beta_{1bp\times 1}, \alpha\beta_{10p\times 1}, \alpha\beta_{21p\times 1},\right.$ $\left.\alpha\beta_{abp\times 1}, \alpha\beta_{a0p\times 1}\right]'$ is the $([a(b+1)] \times p)$ matrix of the interaction effects between A and B^{1}and also control effects for B^0 inside A,
$\mathbf{X}_{AC(B^1)} = \mathbf{1}_r \otimes \mathbf{I}_a \otimes \begin{bmatrix} \mathbf{I}_{bc} \\ \mathbf{0}_{1\times bc} \end{bmatrix}$ is the design matrix for the A×C(B^1) interaction, $\alpha\gamma$ is the $(abc{\times}p)$ matrix of the interaction effects of A×C(B^1),
$\mathbf{X}_1 = \mathbf{I}_{ra} \otimes \mathbf{1}_{bc+1}$ and $\mathbf{I}_N$ are the matrices related to experimental errors $\mathbf{e}_1$ and $\mathbf{e}_2$, respectively,
$\otimes$ denotes Kronecker product of matrices,
$\mathbf{I}_n$ is the $n{\times}n$ identity matrix,
$\mathbf{1}_{n\times 1}$ is the $n{\times}1$ vector of ones,
$\mathbf{0}_{n\times 1}$ is the $n{\times}1$ vector of nulls.

The $\mathbf{e}_{1wp\times 1}$ and $\mathbf{e}_{2dp\times 1}$ columns of the matrices of experimental errors $\mathbf{e}_{1p\times ra}$ and $\mathbf{e}_{2p\times N}$, respectively are mutually, independently distributed as follows:
$\mathbf{e}_{1wp\times 1} \sim N_p(\mathbf{0}, \Sigma_1), w = 1,\ldots, ra;$

$\mathbf{e}_{2dp\times 1} \sim N_p(\mathbf{0}, \Sigma_1),\ d = 1,\ldots, N, \Sigma_1$ and Σ_2 are symmetric positive definite matrices.

The division of the matrix of the μ, β and $\alpha\beta$ parameters in respect to the two forms of the linear model (3) results in block-diagonal sub-matrices occurring in the $\mathbf{X}_{\mathrm{M}}$, $\mathbf{X}_{\mathrm{B}}$, and in $\mathbf{X}_{\mathrm{AB}}$ matrices.

MS ESTIMATORS

In the fixed split plot model, the normal equations for the estimators of the model parameters take the following form [17]:

$$\mathbf{X}'\mathbf{V}^{-1}\mathbf{X}\hat{\Theta} = \mathbf{X}'\mathbf{V}^{-1}\mathbf{Y}',$$

$$\Theta_{q\times p} = \left[\theta_1, \theta_2, \ldots \theta_p\right],$$

$$\mathbf{V} = \Sigma_1 \otimes (\mathbf{I}_{ra} \otimes \mathbf{E}_{bc+1}) + \Sigma_1 \otimes \mathbf{I}_N, \quad (5)$$

where $\mathbf{X}$ is the design matrix, $\mathbf{V}^{-1}$ denotes the inverse of the matrix $\mathbf{V}$ covariance, Θ is the matrix of the fixed parameters. The forms of the parameter estimators in the classical, multivariate split plot design are known [14] and can be easily adapted to the considered model. Taking into account the additional control treatment and the new, two-form model of such a design, we solve the standard equations in order to obtain the parameter estimators. In the considered design we obtain:

$$\boldsymbol{X} =$$

$$\left[\mathbf{X}_M \mid \mathbf{X}_R \mid \mathbf{X}_A \mid \mathbf{X}_B \mid \mathbf{X}_{C(B^1)} \mid \mathbf{X}_{AB} \mid \mathbf{X}_{AC(B^1)}\right],$$

$$\hat{\theta}_{hq\times 1} = [\hat{\mu}_h' \mid \hat{\rho}_h' \mid \hat{\alpha}_h' \mid \hat{\beta}_h' \mid \hat{\gamma}_h' \mid \hat{\alpha\beta}_h'$$
$$\mid \hat{\alpha\gamma}_h']'.$$

In order to obtain the unique solution of the standard equations, we need to add restrictions on the model parameters. Here, we take the following restrictions for each variable $\boldsymbol{Y}_h$:

$$\sum_i \rho_{ih} = 0, \sum_j \alpha_{jh} = 0, \beta_{0h} = 0, \sum_{k=1}^{b} \beta_{kh} = 0,$$

$$\forall k \neq 0 \sum_l \gamma_{l(k)h} = 0, \forall k \neq 0 \sum_j (\alpha\beta)_{jkh} = 0,$$

$$\forall j \sum_{k=1}^{b} \left[c(\alpha\beta)_{jkh} + (\alpha\beta)_{j0h}\right] = 0,$$

$$\forall k \neq 0 \wedge \forall j \sum_{l} (\alpha\gamma)_{jl(k)h} = 0,$$

$$\forall l \sum_{k=1, j=1} (\alpha\gamma)_{jl(k)h} = 0. \qquad (6)$$

We assumed that the vector of the fixed $\beta_{0 p\times 1}$ effect is equal to the null vector because the factor B is not used in the control plots.

HIPOTHESES AND TESTS

We consider a multivariate general linear hypothesis on the equality of some vectors of parameters that correspond to the univariate hypothesis:

$$\mathbf{K}_{d\times q}\Theta_{q\times p}\mathbf{M}_{d\times u} = 0_{d\times u}$$

with the alternative

$$\mathbf{K}_{d\times q}\Theta_{q\times p}\mathbf{M}_{d\times u} \neq 0_{d\times u}, \qquad (7)$$

under the assumption that:

$$\text{rank } (\mathbf{K}) \leq \min \text{ [rank } (\mathbf{X}) \text{ ; d]}.$$

We are interested only in the comparisons of the parameter matrix rows Θ, this assumption corresponds to assuming the **M** matrix equal to identity $\mathbf{I}_p$ matrix. So, the hypothesis focuses the comparisons between the parameters for each variable separately. According to the model experiment, we will test the following hypotheses:

- H_{0Z_T} assuming that the two vectors of the general means μ_1 and μ_0 do not differ efficiently:

$$\mathbf{Z}_T = \mu_1 - \mu_0 = 0,$$

- H_{0A} assuming that the vector of α effects of the A factor is not efficient,
- H_{0B^1} assuming that the non-zero B^1 levels of the B factor do not influence the change of values in the considered variables,
- $H_{0C(B^1)}$ assuming that the γ vector of the $C(B^1)$ factor effects is not efficient,
- H_{0AB} assuming that the $\alpha\beta$ vector of the interaction between $A{\times}B^1$ effects, is not efficient,
- H_{0AC} assuming that the $\alpha\gamma$ vector of the interaction between $A{\times}C(B^1)$ effects, is not efficient,
- $H_{0Z_{AB^1}}$ assuming that the vectors of the effects of the particular levels of the A factor do not change efficiently after application of the non-zero levels of the B^1 factor:

$$\forall j: (\alpha\beta)_{j0} - \frac{1}{b}\sum_{k=1}^{b} (\alpha\beta)_{jk} = 0.$$

The null linear hypothesis $\mathbf{K}\Theta = \mathbf{0}$ concerning particular sources of variation, can be formulated in many ways, we propose assuming the following matrices:

$$\mathbf{K}_{Z_T} = [1 \vdots -1], \mathbf{K}_{A(a-1)\times a} = \left[\mathbf{I}_{(a-1)} \mid -1_{a-1}\right],$$

$$\mathbf{K}_{B^1(b-1)\times b} = \left[\mathbf{I}_{(b-1)} \mid -\mathbf{1}_{b-1}\right],$$

$$\mathbf{K}_{Cb(c-1)\times[b(c-1)+1]} = \left[\mathbf{I}_{b(c-1)} \mid -\mathbf{1}_{b(c-1)}\right],$$

$$\mathbf{K}_{AB^1(a-1)(b-1)\times[(a-1)(b-1)+1]}$$
$$= \left[\mathbf{I}_{(a-1)(b-1)} \mid -\mathbf{1}_{(a-1)(b-1)}\right],$$

$$\mathbf{K}_{AC(a-1)b(c-1)\times[(a-1)b(c-1)+1]}$$
$$= \left[\mathbf{I}_{(a-1)b(c-1)} \mid -\mathbf{1}_{(a-1)b(c-1)}\right],$$

$$\mathbf{K}_{Z_{AB^1}} = \mathbf{I}_a \otimes \left[\frac{-1}{b}\mathbf{1}'_b \mid 1\right], \qquad (8)$$

which will be allocated in the **K** matrix of a multivariate general hypothesis, in an the appropriate place with other **K** elements equal to zero.

For one Y_h, variable the column space C(**X**) of the design matrix **X** is called the estimation space of the linear model. This estimation space is a subspace of R_N; its orthogonal complement in R_N is called the error space. We can subtract the r(**X**) orthogonal subspaces from C(**X**) space, where r(**X**) denotes the rank of **X**, so C(**X**) can be written as a simple sum of the orthogonal subspaces corresponding to the sets of estimable orthogonal linear functions $\lambda'\theta$ of the model parameters. The estimate **l'y** of the estimable function $\lambda'\theta$ is the linear function of the observations. There is one to one correspondence between the estimable $\lambda'\theta$ functions and their best **l'y** estimates (the expected value of **l'y** is equal to $\lambda'\theta$ and **l'y** has the minimum of variance), linearly independent estimable functions have linearly independent best estimates. A set of linear **Ly** functions of the observations carry k degrees of freedom, if we can find k linearly independent functions in the set, and no more, k = r(**L**). Given a set of linear functions of the observations, the square of the **y** projection on the row space R(**L**) of **L** will be called the sum of squares due to the **Ly** set, and its degrees of freedom will be the same as carried by the **Ly** set. The sums of the squares (quadratic forms) arising from the mutually orthogonal sets of functions are called orthogonal sums of squares. According to the division of the C(**X**) estimation space, the sum of squares, arising from the set of all the mutually orthogonal linear functions, can be divided into r(**X**) orthogonal sums of the squares. The **P** matrices occurring in the orthogonal sums of **y'Py** squares are orthogonal projection operators onto appropriate subspaces, **P** = **P'**, **PP'**= **I**.

The univariate analysis of variance can be simply extended to p-variate response variables by replacing the vector of **y** observation in the appropriate quadratic **y'Py** forms by the matrix of **Y** observations [4,15]. In the univariate analysis of variance, the quadratic forms corresponding to the particular null hypothesis for the vectors of parameters in the considered model are given in [10].

The projection operators occurring in these quadratic forms are as follows:

$$\mathbf{P}\mathrm{m} = \frac{1}{ra}\mathbf{E}_{ra} \otimes \begin{bmatrix} \frac{1}{bc}\mathbf{E}_{bc} & \mathbf{0}_{bc} \\ \mathbf{0}'_{bc} & 1 \end{bmatrix},$$

$$\mathbf{P}_R = \frac{1}{a(bc+1)}\left(\mathbf{I}_r - \frac{1}{r}\mathbf{E}_r\right) \otimes \mathbf{E}_{a(bc+1)},$$

$$\mathbf{P}_A = \frac{1}{r(bc+1)}\mathbf{E}_r \otimes \left(\mathbf{I}_a - \frac{1}{a}\mathbf{E}_a\right) \otimes \mathbf{E}_{(bc+1)},$$

$$\mathbf{P}_\mathrm{B} = \frac{1}{ra}\mathbf{E}_{ra} \otimes$$

$$\left(\begin{bmatrix} \mathbf{I}_b \otimes \frac{1}{c}\mathbf{E}_c & \mathbf{0}_{bc} \\ \mathbf{0}'_{bc} & 1 \end{bmatrix} - \frac{1}{bc+1}\mathbf{E}_{bc+1}\right),$$

$$\mathbf{P}_{AB} = \frac{1}{r}\mathbf{E}_r \otimes \left(\mathbf{I}_a - \frac{1}{a}\mathbf{E}_a\right) \otimes$$

$$\left(\begin{bmatrix} \mathbf{I}_b \otimes \frac{1}{c}\mathbf{E}_c & \mathbf{0}_{bc} \\ \mathbf{0}'_{bc} & 1 \end{bmatrix} - \frac{1}{bc+1}\mathbf{E}_{bc+1}\right),$$

$$\mathbf{P}_1 = \frac{1}{bc+1}\left(\mathbf{I}_r - \frac{1}{r}\mathbf{E}_r\right) \otimes$$

$$\left(\mathbf{I}_a - \frac{1}{a}\mathbf{E}_a\right) \otimes \mathbf{E}_{bc+1}, \qquad (9)$$

where: $\mathbf{E}_n$ is the $n{\times}n$ matrix of ones, $\mathbf{E}_{n\times m}$ is the $n \times m$ matrix of ones. In the multivariate analysis of variance, the sums of products can be obtained in the same way.

With respect to the control treatment and two grand means the following additional sub-spaces have been separated:

- C($\mathbf{X}_{Z_T}$) for the $\mathbf{Z}_T = \mu_{1\mathrm{h}} - \mu_{0\mathrm{h}}$ contrasts,
- C($\mathbf{X}_{B^1}$) for the non-zero levels of the B factor,
- C($\mathbf{X}_{AB^1}$) for the interaction between A and non-zero B^1 levels of B,
- C($\mathbf{X}_{Z_{AB}}$) for the

$$\mathbf{Z}_{AB} = (\alpha\beta)_{j0} - \frac{1}{b}\sum_{k=1}^{b}(\alpha\beta)_{jk} \text{ contrasts.}$$

The corresponding projection operators $\mathbf{P}_{B^1}$, $\mathbf{P}_{Z_T}$, $\mathbf{P}_{AB^1}$ and $\mathbf{P}_{Z_{AB}}$, for the additional sub- spaces, take the following forms:

$$\mathbf{P}_{B_1} = \frac{1}{ra}\mathbf{E}_{ra} \otimes \begin{bmatrix} \mathbf{I}_{bc} - \frac{1}{bc}\mathbf{E}_{bc} & \mathbf{0}_{bc} \\ \mathbf{0}'_{bc} & 0 \end{bmatrix},$$

$$\mathbf{P}_{Z_T} = \frac{1}{ra}\mathbf{E}_{ra} \otimes \frac{1}{bc+1}\begin{bmatrix} \frac{1}{bc}\mathbf{E}_{bc} & \mathbf{-1}_{bc} \\ \mathbf{-1}'_{bc} & bc \end{bmatrix},$$

$$\mathbf{P}_{AB_1} = \frac{1}{r}\mathbf{E}_r \otimes \left(\mathbf{I}_a - \frac{1}{a}\mathbf{E}_a\right) \otimes$$

$$\begin{bmatrix} \mathbf{I}_{bc} - \frac{1}{bc}\mathbf{E}_{bc} & \mathbf{0}_{bc} \\ \mathbf{0}'_{bc} & 0 \end{bmatrix},$$

$$\mathbf{P}_{Z_{AB^1}} = \frac{1}{r}\mathbf{E}_r \otimes \left(\mathbf{I}_a - \frac{1}{a}\mathbf{E}_a\right) \otimes$$

$$\frac{1}{bc+1}\begin{bmatrix} \frac{1}{bc}\mathbf{E}_{bc} & \mathbf{-1}_{bc} \\ \mathbf{-1'}_{bc} & bc \end{bmatrix}. \qquad (10)$$

It will be pointed out that the particular quadratic forms in the univariate analysis of variance have an independent non-central χ^2 distribution [10]. The structure of the covariance matrix Σ does not influence this distribution. The appropriate sums of the products for the hypotheses given by matrices (8) can be calculated as the following products of matrices:

$$\mathbf{H}_R = \mathbf{YP}_R\mathbf{Y'}, \mathbf{H}_A = \mathbf{YP}_A\mathbf{Y'}, \mathbf{H}_B = \mathbf{YP}_B\mathbf{Y'},$$

$$\mathbf{H}_C = \mathbf{YP}_C\mathbf{Y'}, \mathbf{H}_{B^1} = \mathbf{YP}_{B^1}\mathbf{Y'},$$

$$\mathbf{H}_{Z_T} = \mathbf{YP}_{Z_T}\mathbf{Y'}, \mathbf{H}_{AB} = \mathbf{YP}_{AB}\mathbf{Y'},$$

$$\mathbf{H}_{AB^1} = \mathbf{YP}_{AB^1}\mathbf{Y'}, \mathbf{H}_{Z_{AB^1}} = \mathbf{YP}_{Z_{AB^1}}\mathbf{Y'},$$

$$\mathbf{H}_{AC} = \mathbf{YP}_{AC}\mathbf{Y'}, \qquad (11)$$

and the sums of products for experimental errors:

$$\mathbf{G}_{E1} = \mathbf{YP}_1\mathbf{Y'},$$

$$\mathbf{G}_{E2} = \mathbf{YY'} - \mathbf{H}_R - \mathbf{H}_A - \mathbf{H}_B - \mathbf{H}_C - \mathbf{H}_{AB} - \mathbf{H}_{AC} - \mathbf{G}_{E1}.$$

The W_s function based on the maximum eigenvalue λ_s of the matrix type $\mathbf{HG}^{-1}$ will be used as the test function, $W_s = \frac{\lambda_s}{1+\lambda_s}$ [16]. When the null hypothesis is true, then the parameters of the distribution of the variable W_s are s, m, and n calculated according to the formulas [15]:

$$s = \min\,[r(\mathbf{K}), r(\mathbf{M})],\ m = \frac{|r(\mathbf{K}) - r(\mathbf{M})| - 1}{2},$$

$$n = \frac{N - r(\mathbf{X}) - p - 1}{2},\ N = ra(bc+1). \qquad (12)$$

For s=1 random variable $F = \frac{n+1}{m+1}\frac{W}{1-W} = \frac{n+1}{m+1}\lambda$ has F distribution with $\nu_1 = 2m+2$ and $\nu_2 = 2n+2$ degrees of freedom. The null hypothesis will be rejected on the level α of significance if $W_s > W_{\alpha;s,m.,n}$, where $W_{\alpha;s,m.,n}$ is above a 100α -percent critical value from the Pillais tables or from the Hecks monograms.

The hypothesis for the $\mathbf{Z}_T$ contrasts $\mathbf{H}_{Z_{T0}}: \mu_1 - \mu_0 = 0$ on the equality of the means of the component models can be treated as a joined comparison of the tested factors with the control levels. In this case, we obtain an answer to the important question about the influence of the experimental factors on a given characteristics. Further inference concerning the comparison of particular means with the

T a b l e 1. The summarised yields of sugar beet (t ha^{-1})

Irradiation	Varieties of sugar beet							
	Colibri		Evita		Kawetina		Maria	
	leaves	roots	leaves	roots	leaves	roots	leaves	roots
Lamp 1	116.5	173.7	138.8	147.7	161.5	166.1	139.5	154.1
Lamp 2	122.6	190.1	134.7	161.7	164.5	173.0	148.5	173.2
Lamp 3	131.8	193.6	133.7	156.8	151.6	178.6	145.7	168.9
Lamp 4	132.8	190.2	125.5	151.0	141.7	180.0	146.5	167.8
Laser 1	109.9	173.9	131.2	156.6	154.0	181.6	141.8	172.6
Laser 2	105.2	182.1	130.3	140.4	159.5	184.4	132.2	161.6
Laser 3	107.9	175.1	128.4	165.9	167.7	171.0	153.5	169.8
Laser 4	132.9	181.8	124.5	161.9	165.2	167.3	140.6	158.2
Magnet. Field 1	127.3	192.2	128.1	141.7	144.1	164.2	144.6	161.6
Magnet. Field 2	129.7	188.2	128.5	160.4	155.5	177.5	142.8	157.9
Magnet. Field 3	129.0	171.2	119.2	154.4	156.8	174.0	141.8	131.8
Magnet. Field 4	132.6	172.2	127.7	143.2	152.0	174.5	129.8	164.7
Control	109.3	168.7	140.1	160.3	148.3	168.0	93.4	133.0

control mean can be carried out using simultaneous multivariate multiple comparisons [16].

NUMERICAL EXAMPLE

The experiment, conducted according to the split- plot scheme was arranged in 3 blocks ($r = 3$). The varieties of sugar beet (A, $a = 4$) have been compared, in relation to the seed stimulation with a lamp, laser and an electromagnetic (B, $b = 3$) field and different doses of this stimulation (C, $c = 4$). The $a\,r$ control plots have been sown with the seeds, which had not been subjected to stimulation. Doses of C irradiation were treated as a factor occurring inside B stimulation, since a lamp impulse is not comparable to a laser impulse or time of the magnetic field influence. Table 1 presents the summarised yields from 3 blocks.

The following hypotheses were tested:

1. H_{Z_T}: About the equality of the vectors of the grand means for the control plots and for the plots, where the factors have been used. The null hypothesis assumes, that the tested factors jointly do not influence changes in the values of the considered variables.
2. H_A: About the equality of the vectors of the variety effects.
3. H_{B^1}: About the equality of the vectors of the lamp, laser and electromagnetic field effects. The null hypothesis assumes that these three levels of the B^1 factor: lamp, laser and electromagnetic field affect the values of the considered variables in the same way.
4. H_C: About the equality of the vectors of the impulse effects. The null hypothesis assumes that the considered impulse numbers of the factors: lamp, laser and electromagnetic field affect the values of the considered variables in the same way.
5. H_{AB^1}: About the equality of the vectors of the interaction effects between variety A and stimulation B. The null hypothesis assumes that these three factors: lamp, laser and electromagnetic field influence the values of the variables of the considered varieties of the sugar beet in the same way.
6. $H_{Z_{AB}}$: About the equality of the vectors of changes of the effects for the particular varieties caused by the seeds stimulation.

The sums of products (11) for the null hypotheses given by matrices (8) and covariance matrices for errors are the following:

$$G_{E1} = \begin{bmatrix} 123.55 & -54.24 \\ & 1071.38 \end{bmatrix},$$

$$H_R = \begin{bmatrix} 1101.44 & -55.50 \\ & 1343.44 \end{bmatrix},$$

$$H_A = \begin{bmatrix} 2648.87 & 60.40 \\ & 2020.74 \end{bmatrix},$$

$$G_{E2} = \begin{bmatrix} 588.52 & 36.47 \\ & 1789.59 \end{bmatrix},$$

$$H_B = \begin{bmatrix} 313.33 & 253.23 \\ & 240.83 \end{bmatrix},$$

$$H_C = \begin{bmatrix} 155.82 & 103.49 \\ & 316.49 \end{bmatrix},$$

$$H_{B^1} = \begin{bmatrix} 59.00 & 68.79 \\ & 107.07 \end{bmatrix},$$

$$H_T = \begin{bmatrix} 254.33 & 184.44 \\ & 133.76 \end{bmatrix},$$

$$H_{AB} = \begin{bmatrix} 854.13 & 462.28 \\ & 308.41 \end{bmatrix},$$

$$H_{AB^1} = \begin{bmatrix} 324.74 & 137.67 \\ & 104.70 \end{bmatrix},$$

$$H_Z = \begin{bmatrix} 529.40 & 324.61 \\ & 203.71 \end{bmatrix},$$

$$H_{AC} = \begin{bmatrix} 254.33 & 184.44 \\ & 133.76 \end{bmatrix}.$$

The inverses of the covariance matrices for the errors are the following:

$$G_{E1}^{-1} = \begin{bmatrix} 0.0083 & 0.00042 \\ & 0.00095 \end{bmatrix},$$

$$G_{E2}^{-1} = \begin{bmatrix} 0.0017 & -0.000035 \\ & 0.00056 \end{bmatrix}.$$

According to the one variable analysis of variance, we will test the hypotheses for each source of variation separately. The maximum eigenvalue λ_s of the matrices HG^{-1} will be used as the test function.

The hypothesis for the $\mathbf{Z}_T$ contrast was verified by using the F statistic. For s=1 random variable $F = \frac{n+1}{m+1}\frac{W}{1-W}$ has F distribution with $v_1 = 2m + 2$=1 and $v_2 = 2n + 2$=101 d.f.

The W_0 values exceed the critical $W_{0.01}$ values (Table 2), which resultsed in rejecting the null hypotheses. We have got not an answer to the question which levels of factors and which variables or else which combinations of variables and factors are responsible for rejecting the null hypotheses. A more detailed inference will be possible after using the multiple comparisons. The calculations were made by using a computer Maple program, while a program for the total statistical analysis of the experiments with the control treatment according to the presented theory is under preparation now.

Table 2. Analysis of variance for the yields of the sugar beet

Sources of variation	r(**K**)	s	m	Max λ_s	n	W_0	$W_{0.01}$
Blocks (R)	2	2	-0.5	1.26	49.5	0.56	0.115
Variety (A)	3	2	0	22.03	49.5	0.96	0. 130
Error I	6	2	1.5	0.63			
Irradiation B	3	2	0		49.5	0.39	0.130
Inside: 1. Contrast Z_T	1	1	-0.5	0.49	49.5	33.33	7.21
2. Irradiation B_1	2	2	-0.5	0.15	49.5	0.13	0.115
Dosage of irradiation C	9	2	3	0.32	49.5	0.24	0.225
Interaction (AxB)	9	2	1.5	1.56	49.5	0.61	0.195
Inside: 1. Contrasts $\mathbf{Z}_{AB}$	3	2	0	0.99	49.5	0.50	0.130
2. AxB^1	6	2	1.5	0.58	49.5	0.37	0.195
Interaction AxC	27	2	12	1.01	49.5	0.50	0.350
Error II	96						

CONCLUSIONS

1. A new form of the model (3) with two sub-models enables the analysis of variance for the tested factors without the control, solely on the basis of the model (2).

2. The analysis of variance for the tested factors, conducted only on the basis of the model (2), gives conclusions comparable with the conclusions from another experiments with the same factors, but without control.

3. Model (3) gives a better visualisation of the experiment than the traditional model.

4. In this new form of the model, control effects are separated in the beginning.

5. In the joined model (3), we have got, $a\ r$ degrees of freedom more than in (2) for each variable. Additional degrees of freedom will be used for the comparison of the tested factors with the control, together or separately.

6. The new formula of the model is especially useful, when control treatment means that no treatment is applied to the experimental units, so effects are equal to zero.

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