Abstract. The paper describes multivariate analysis of variance in the split plot design when the so-called control treatment $B_0$ is allocated to an additional subplot. The factor $B^1$ allocated to small plots is subject to the two-stage classification, where the levels of the second stage dependent on the levels of the first stage. This dependence is treated as a hierarchical classification of two factors: $B^1$ - the first stage levels and $C$ (inside $B^1$) - the second stage levels. A linear model of the experiment consisting of two forms is used, one form for the plots where all the factors are present and the other for control plots. Besides that, for the sake of better visualisation of the experiment, the component models are based on two different vectors of the grand means. Tests of hypotheses for the comparison with the control are presented. The discussed theory is illustrated by an agrophysical experiment conducted according to such a design.

Keywords: contrasts, control object, hierarchical classification, multivariate analysis of variance, split plot design


INTRODUCTION

Split plot designs are widely used in field experiments. The problem of the comparison of group objects with control objects often occurs in variety, fertilization and feeding experiments. Recently, agrophysical experiments based on split-plot design have been carried out. These experiments need new elaboration of statistical analysis with respect to the control object and few-stage classification of one factor. The literature on the subject of block designs for univariate analysis of variance is extensive. The problem of one or more control objects has been considered with respect to development of experimental design [2,5,11,13] and with respect to statistical analysis of such designs [1,3,9]. Beside that, this problem was considered in the light of the assumption of the equivalence in alternative hypothesis for the contrasts between test objects and the control object [6]. Another important problem, is determination of sample sizes for the comparison of $k$ treatments against a control [7,8,12].

In this paper a multivariate analysis of variance, for a new model of experiment with two factors $A$ and $B^1$ and with an additional control object $B_0$ set up according to the split plot design is considered [10]. An experiment, in which the levels of the $A$ factor ($A_j$, $j=1,2,...,a$) were randomly allocated to $a$ whole-plots. Inside each level $A_j$ of the factor $A$, $bc$ levels of the second $B^1$ factor and one control $B_0$ object were allocated to $bc + 1$ plots. The $bc$ levels of $B^1$ are obtained, as a result of a two stage classification, in the following way: there are $b$ first stage levels $B_k$ ($k=1,2,...,b$) of $B^1$ and $c$ second stage levels $C_l(B_k)$ ($l=1,2,...,c$) inside each level $B_k$ of $B^1$. These $c$ levels will be treated as the levels of the third factor $C$ inside $B_1$. Together, the factor $B^1$ and the control object $B_0$ are denoted as $B$. A hierarchical classification was adopted because of the
character of the $C_h B_{k}$ levels of the C factor that
dependent on the $B_{k}$ levels of the $B^1$ factor.
Additional fixed effects of C and appropriate
interaction effects were extended a classical
linear model for the split plot design. The effects
of blocks are treated as fixed effects. Random
errors are the same as in the classical model,
since the experiment was arranged according to
the scheme of the split plot design with two
factors. The goal of this paper is to present the
multivariate analysis of variance for a new form
of the model of such an experiment given by
Kuna-Broniowska and Przybysz [10]. The
matrices with appropriate sums of products for a
particular null hypotheses are based on the
quadratic forms obtained in univariate analysis
of variance [10]. The discussed theory is widely
illustrated by an example of an experiment
conducted in Lublin, where a factor was subjected
to two-stage classification with a relation be-
tween the first and second levels.

MODEL

The observations for the $h$-th variable of
the experiment conducted in $r$ blocks, can be
described according to one of the two linear
models:

$$
y_{ij0h} = \mu_{1h} + \rho_{ih} + \alpha_{jh} + e_{ijh} + \beta_{0h} + e_{ij00h}
$$

(1)

for the control plots, where the $B^1$ and C factors
are not present,

$$
y_{ijklh} = \mu_{1h} + \rho_{ih} + \alpha_{jh} + e_{ijh} + \beta_{kh} + e_{ijkl0h} + (\alpha \beta)_{ijkl0h}
$$

(2)

for the other plots.

The joined model of the complete experi-
takes the following form:

$$
y_{ijklh} = \begin{bmatrix}
\mu_{1h} + \rho_{ih} + \alpha_{jh} + e_{ijh} + \beta_{kh} + e_{ijkl0h} \\
\gamma \ell(k)h + (\alpha \beta)_{ijklh} \\
+ e_{ijklh} & \text{for } k \neq 0, \ell \neq 0 \\
\mu_{0h} + \rho_{ih} + \alpha_{jh} + e_{ijh} + \beta_{0h} \\
+ (c \gamma)_{ijklh} & \text{for } k = 0, \ell = 0
\end{bmatrix}
$$

(3)

where: $h = 1, \ldots, p$; $i = 1, \ldots, r$; $j = 1, \ldots, a$; $k = 0, \ldots, b$;
$l = 0, \ldots, c$; $\mu$ and $\mu_0$ are general means, $\rho$, $\alpha$, and
$\beta$ are the effect of the $i$-th block, $\alpha_{jh}$ is the effect of the
$j$-th level of A, $\beta_{kh}$ is the effect of the $k$-th level of
$B^1$, $\gamma \ell(k)h$ is the effect of the $l$-th level of the C
factor inside the $k$-th level of $B^1$, $(\alpha \beta)_{ijklh}$ and
$(c \gamma)_{ijklh}$ are relevant interaction effects,
$e_{ijklh}$ and $e_{ijkl0h}$ are experimental errors.

In the model (3) there are two groups of
general means, namely: $\mu_{1h}$ and $\mu_{0h}$.
This point of view was adopted because of future
estimators and analyses.

Using the matrix notation and assuming a
traditional data arrangement, namely according
to the blocks, next the A, B factors and finally a
C model (3) can be written:

$$
Y_{N,p} = X_M \mu_{2,p} + X_R \rho_{r,p} + X_A \alpha_{a,p} + X_C \beta_{c,p} + X_{AB} \gamma_{bc,p} + X_{AC} \gamma_{ac,p} + Y_{N} e_{2N,p}
$$

(4)

where: $N = ra (bc+1)$, $Y^*$ is the $(N \times p)$ matrix of
observations,

$$
X_M = I_{rad} \otimes \begin{bmatrix} 1_{bc \times 1} & 0_{bc \times 1} \\ 0_{bc \times 1} & 1_{bc \times 1} \end{bmatrix}
$$

is the design
matrix for general means occurring by

$$
\mu = \begin{bmatrix} \mu_{11,p} \\ \mu_{01,p} \end{bmatrix}
$$

$X_R = I_r \otimes I_{a(bc+1) \times 1}$ is the design matrix for the
R blocks, $\rho$ is the $(r \times p)$ matrix of fixed
effects of blocs,
$X_A = I_{ra} \otimes I_a \otimes I_{bc+1}$ is the design matrix for the A factor, $\alpha$ is the $(a \times p)$ matrix of fixed effects of A factor, $X_B = I_{rax1} \otimes \begin{bmatrix} I_b \otimes I_{cx1} & 0_{bcx1} \\ 0_{1xb} & 1 \end{bmatrix}$ is the design matrix for the B factor, $\beta_{(b+1)p} = \begin{bmatrix} \beta_{1p1}, \ldots, \beta_{bp1}, \beta_{0p1} \end{bmatrix}$ is the $(b+1) \times p$ matrix of fixed effects of the B factor, $X_{C(B)} = I_{ra} \otimes \begin{bmatrix} I_{bc} & 0_{1xb} \\ 0_{1xbc} & 1 \end{bmatrix}$ is the design matrix for the C(B) factor, $\gamma$ is the $(bc \times p)$ matrix of the fixed effects of the C factor, $X_{AB} = I_{rax1} \otimes I_a \otimes \begin{bmatrix} I_b \otimes I_{cx1} & 0_{bcx1} \\ 0_{1xb} & 1 \end{bmatrix}$ is the design matrix for the AB interaction, $\alpha\beta = \begin{bmatrix} \alpha\beta_{11p1}, \ldots, \alpha\beta_{1bp1}, \alpha\beta_{00p1} \end{bmatrix}$ is the $(a(b+1)) \times p$ matrix of the interaction effects between A and B and also control effects for B inside A, $X_{AC(B)} = I_r \otimes I_a \otimes \begin{bmatrix} I_{bc} \\ 0_{1xbc} \end{bmatrix}$ is the design matrix for the A\!C(B) interaction, $\gamma\alpha\beta = \begin{bmatrix} \gamma\alpha\beta_{11p1}, \ldots, \gamma\alpha\beta_{1bp1}, \gamma\alpha\beta_{00p1} \end{bmatrix}$ is the $(a(b+1)) \times p$ interaction matrix of the interaction effects of A\!C(B').

The $\otimes$ denotes Kronecker product of matrices, $I_n$ is the $n \times n$ identity matrix, $I_{ra}$ is the $a \times a$ vector of ones, $0_{a \times 1}$ is the $a \times 1$ vector of nulls.

The $e_{1pwx}$ and $e_{2px}$ columns of the matrices of experimental errors $e_{1pwx}$ and $e_{2px}$, respectively, are mutually, independently distributed as follows:

- $e_{1pwx} \sim N_p(0, \Sigma_1), w = 1, \ldots, ra$;
- $e_{2px} \sim N_p(0, \Sigma_1), d = 1, \ldots, N, \Sigma_1$ and $\Sigma_2$ are symmetric positive definite matrices.

The division of the matrix of the $\mu, \beta$ and $\alpha\beta$ parameters in respect to the two forms of the linear model (3) results in block-diagonal sub-matrices occurring in the $X_M$, $X_B$, and in $X_{AB}$ matrices.

**MS ESTIMATORS**

In the fixed split plot model, the normal equations for the estimators of the model parameters take the following form [17]:

$$X'V^{-1}X\hat{\Theta} = X'V^{-1}Y,'$$

$$\Theta_{q,p} = \begin{bmatrix} \theta_{1}, \theta_{2}, \ldots, \theta_{p} \end{bmatrix},$$

$$V = \Sigma_1 \otimes \left( I_{w} \otimes E_{bc+1} \right) + \Sigma_1 \otimes I_{N},$$

where $X$ is the design matrix, $V^{-1}$ denotes the inverse of the matrix $V$ covariance, $\Theta$ is the matrix of the fixed parameters. The forms of the parameter estimators in the classical, multivariate split plot design are known [14] and can be easily adapted to the considered model. Taking into account the additional control treatment and the new, two-form model of such a design, we solve the standard equations in order to obtain the parameter estimators. In the considered design we obtain:

$$X = \begin{bmatrix} X_M \mid X_B \mid X_A \mid X_{C(B')} \mid X_{AB} \mid X_{AC(B')} \end{bmatrix}.$$
\[ \forall k \neq 0 \land \forall j \sum_{l} (\alpha y)_{jl(k)h} = 0, \]
\[ \forall l \sum_{k=1, j} (\alpha y)_{jl(k)h} = 0. \]  
(6)

We assumed that the vector of the fixed \( \beta_{0\uparrow} \) effect is equal to the null vector because the factor B is not used in the control plots.

**Hypotheses and Tests**

We consider a multivariate general linear hypothesis on the equality of some vectors of parameters that correspond to the univariate hypothesis:

\[ K_{d\alpha q} \Theta_{q\alpha p} M_{d\alpha u} = 0_{d\alpha u} \]

with the alternative

\[ K_{d\alpha q} \Theta_{q\alpha p} M_{d\alpha u} \neq 0_{d\alpha u}, \]  
(7)

under the assumption that:

\[ \text{rank (} K \text{)} \leq \min [\text{rank (} X \text{)} ; d]. \]

We are interested only in the comparisons of the parameter matrix rows \( \Theta \), this assumption corresponds to assuming the \( M \) matrix equal to identity \( I_p \) matrix. So, the hypothesis focuses the comparisons between the parameters for each variable separately. According to the model experiment, we will test the following hypotheses:

- \( H_{0Z} \) assuming that the two vectors of the general means \( \mu_1 \) and \( \mu_0 \) do not differ efficiently:

\[ Z_T = \mu_1 - \mu_0 = 0, \]

- \( H_{0A} \) assuming that the vector of \( \alpha \) effects of the A factor is not efficient,

- \( H_{0B} \) assuming that the non-zero \( B \) levels of the B factor do not influence the change of values in the considered variables,

- \( H_{0C(B^1)} \) assuming that the \( \gamma \) vector of the \( C(B^1) \) factor effects is not efficient,

- \( H_{0AB} \) assuming that the \( \alpha \beta \) vector of the interaction between \( A-B^1 \) effects, is not efficient,

- \( H_{0AC} \) assuming that the \( \alpha \gamma \) vector of the interaction between \( A-C(B^1) \) effects, is not efficient,

- \( H_{0Z AB} \) assuming that the vectors of the effects of the particular levels of the A factor do not change efficiently after application of the non-zero levels of the \( B^1 \) factor:

\[ \forall j \sum_{k=1}^{b} (\alpha y)_{j0} - \frac{1}{b} \sum_{k=1}^{b} (\alpha y)_{jk} = 0. \]

The null linear hypothesis \( K \Theta = 0 \) concerning particular sources of variation, can be formulated in many ways, we propose assuming the following matrices:

\[ K_{Z_a} = [1-1], \]
\[ K_{A(a-1)c} = [1_{(a-1)} - 1_{a-1}], \]
\[ K_{B(b-1)c} = [1_{(b-1)} - 1_{b-1}], \]
\[ K_{C(b-1)c} = [1_{(b-1)} - 1_{b-1}], \]
\[ K_{Z_{AB}((a-1)(b-1))c} = [1_{(a-1)(b-1)} - 1_{(a-1)(b-1)+1}], \]
\[ K_{A(a-1)(b-1)c} = [1_{(a-1)(b-1)} - 1_{(a-1)(b-1)+1}], \]
\[ K_{AC(a-1)b(c-1)c} = [1_{(a-1)b(c-1)} - 1_{(a-1)b(c-1)+1}], \]
\[ K_{Z_{AC}((a-1)b c-1)c} = [1_{(a-1)b c-1} - 1_{(a-1)b c-1+1}], \]
\[ K_{Z_{AB}((a-1)b c-1)c} = [1_{(a-1)b c-1} - 1_{(a-1)b c-1+1}], \]
\[ K_{Z_{AC}((a-1)b c-1)c} = [1_{(a-1)b c-1} - 1_{(a-1)b c-1+1}], \]

which will be allocated in the \( K \) matrix of a multivariate general hypothesis, in an the appropriate place with other \( K \) elements equal to zero.

For one \( Y_{bh} \) variable the column space \( C(X) \) of the design matrix \( X \) is called the estimation space of the linear model. This estimation space is a subspace of \( R_N \); its orthogonal complement in \( R_N \) is called the error space. We can subtract the \( r(X) \) orthogonal subspaces from \( C(X) \) space, where \( r(X) \) denotes the rank of \( X \), so \( C(X) \) can be written as a simple sum of the orthogonal subspaces corresponding to the sets of estimable orthogonal linear functions \( \lambda' \theta \) of the model parameters. The estimate \( \gamma' \) of the estimable function \( \lambda' \theta \) is the linear function of the observations. There is one to one correspondence between the estimable \( \lambda' \theta \) functions and...
their best $\Gamma y$ estimates (the expected value of $\Gamma y$ is equal to $\lambda \psi$ and $\Gamma y$ has the minimum of variance), linearly independent estimable functions have linearly independent best estimates.

A set of linear $\mathbf{L}_y$ functions of the observations carry $k$ degrees of freedom, if we can find $k$ linearly independent functions in the set, and no more, $k = r(L)$. Given a set of linear functions of the observations, the square of the $\mathbf{L}_y$ projection on the row space $\mathbf{R}(\mathbf{L})$ of $\mathbf{L}$ will be called the sum of squares due to the $\mathbf{L}_y$ set, and its degrees of freedom will be the same as carried by the $\mathbf{L}_y$ set. The sums of the squares (quadratic forms) arising from the mutually orthogonal sets of functions are called orthogonal sums of squares. According to the division of the $C(X)$ estimation space, the sum of squares, arising from the mutually orthogonal linear functions, can be divided into $r(X)$ orthogonal sums of the squares. The $P$ matrices occurring in the orthogonal sums of $y'Py$ squares are orthogonal projection operators onto appropriate subspaces, $P = P^*, PP^* = I$.

The univariate analysis of variance can be simply extended to $p$-variate response variables by replacing the vector of $y$ observation in the appropriate quadratic $y'Py$ forms by the matrix of $Y$ observations [4,15]. In the univariate analysis of variance, the quadratic forms corresponding to the particular null hypothesis for the vectors of parameters in the considered model are given in [10].

The projection operators occurring in these quadratic forms are as follows:

$$P_m = \frac{1}{ra} \mathbf{E}_{ra} \otimes \begin{bmatrix} \frac{1}{bc} \mathbf{E}_{bc} & 0_{bc} \\ 0_{bc} & 1 \end{bmatrix},$$

$$P_R = \frac{1}{a(bc+1)} \left( \mathbf{I}_a - \frac{1}{r} \mathbf{E}_r \right) \otimes \mathbf{E}_{a(bc+1)},$$

$$P_A = \frac{1}{r(bc+1)} \mathbf{E}_r \otimes \left( \mathbf{I}_a - \frac{1}{a} \mathbf{E}_a \right) \otimes \mathbf{E}_{(bc+1)},$$

$$P_B = \frac{1}{ra} \mathbf{E}_{ra} \otimes \begin{bmatrix} \mathbf{I}_b \otimes \frac{1}{c} \mathbf{E}_c & 0_{bc} & 0_{bc} \\ 0_{bc} & 1_{bc+1} \end{bmatrix},$$

$$P_{AB} = \frac{1}{r} \mathbf{E}_r \otimes \left( \mathbf{I}_a - \frac{1}{a} \mathbf{E}_a \right) \otimes \mathbf{E}_{bc+1},$$

$$P_{1} = \frac{1}{bc+1} \left( \mathbf{I}_r - \frac{1}{r} \mathbf{E}_r \right) \otimes \left( \mathbf{I}_a - \frac{1}{a} \mathbf{E}_a \right) \otimes \mathbf{E}_{bc+1},$$

where: $E_n$ is the $n \times n$ matrix of ones, $E_{mon}$ is the $n \times m$ matrix of ones. In the multivariate analysis of variance, the sums of products can be obtained in the same way.

With respect to the control treatment and two grand means the following additional subspaces have been separated:
- $C(X_{Z^1})$ for the $Z_T = \mu_{1b} - \mu_{0b}$ contrasts,
- $C(X_{B^1})$ for the non-zero levels of the $B$ factor,
- $C(X_{Z_{AB}^1})$ for the interaction between $A$ and non-zero $B^1$ levels of $B$,
- $C(X_{Z_{AB}})$ for the $Z_{AB} = (a\beta_j)_{b=1}^b - \frac{1}{b} \sum b (a\beta_j)_{jk}$ contrasts.

The corresponding projection operators $P_{Z^1}, P_{Z_T}, P_{B^1}$ and $P_{Z_{AB}}$; for the additional sub- spaces, take the following forms:

$$P_{B^1} = \frac{1}{rd} \mathbf{E}_{ra} \otimes \begin{bmatrix} \mathbf{I}_{bc} - \frac{1}{bc} \mathbf{E}_{bc} & 0_{bc} \\ 0_{bc} & 0 \end{bmatrix},$$

$$P_{Z_T} = \frac{1}{rd} \mathbf{E}_{ra} \otimes \left( \frac{1}{bc+1} \begin{bmatrix} \mathbf{I}_{bc} & -\mathbf{I}_{bc} \\ -\mathbf{I}_{bc} & bc \end{bmatrix}, \right.$$

$$P_{Z_{AB}} = \frac{1}{rd} \mathbf{E}_{ra} \otimes \mathbf{E}_{a(bc+1)} \otimes \begin{bmatrix} \mathbf{I}_{bc} & 0_{bc} & 0_{bc} \\ 0_{bc} & 0 & 0 \end{bmatrix}.$$
It will be pointed out that the particular quadratic forms in the univariate analysis of variance have an independent non-central \( \chi^2 \) distribution [10]. The structure of the covariance matrix \( \Sigma \) does not influence this distribution. The appropriate sums of the products for the hypotheses given by matrices (8) can be calculated as the following products of matrices:

\[
P_{Z_{ab}1} = \frac{1}{r} E_r \otimes \left( I_a - \frac{1}{d} E_a \right) \otimes \frac{1}{bc+1} \begin{bmatrix}
\frac{1}{bc} E_{bc} & -1_{bc} \\
-1_{bc} & bc
\end{bmatrix},
\]

(10)

and the sums of products for experimental errors:

\[
G_{E1} = YP_Y Y',
\]

\[
G_{E2} = YY' - HR - HA - HB - HC - HAB - HAC - GE1.
\]

Table 1. The summarised yields of sugar beet (t ha\(^{-1}\))

<table>
<thead>
<tr>
<th>Irradiation</th>
<th>( \text{Colibri} )</th>
<th>( \text{Evita} )</th>
<th>( \text{Kawetina} )</th>
<th>( \text{Maria} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lamp 1</td>
<td>116.5</td>
<td>173.7</td>
<td>138.8</td>
<td>147.7</td>
</tr>
<tr>
<td>Lamp 2</td>
<td>122.6</td>
<td>190.1</td>
<td>134.7</td>
<td>161.7</td>
</tr>
<tr>
<td>Lamp 3</td>
<td>131.8</td>
<td>193.6</td>
<td>133.7</td>
<td>156.8</td>
</tr>
<tr>
<td>Lamp 4</td>
<td>132.8</td>
<td>190.2</td>
<td>125.5</td>
<td>151.0</td>
</tr>
<tr>
<td>Laser 1</td>
<td>109.9</td>
<td>173.9</td>
<td>131.2</td>
<td>156.6</td>
</tr>
<tr>
<td>Laser 2</td>
<td>105.2</td>
<td>182.1</td>
<td>130.3</td>
<td>140.4</td>
</tr>
<tr>
<td>Laser 3</td>
<td>107.9</td>
<td>175.1</td>
<td>128.4</td>
<td>165.9</td>
</tr>
<tr>
<td>Laser 4</td>
<td>122.9</td>
<td>181.8</td>
<td>124.5</td>
<td>161.9</td>
</tr>
<tr>
<td>Magn. Field 1</td>
<td>127.3</td>
<td>192.2</td>
<td>128.1</td>
<td>141.7</td>
</tr>
<tr>
<td>Magn. Field 2</td>
<td>129.7</td>
<td>188.2</td>
<td>128.5</td>
<td>160.4</td>
</tr>
<tr>
<td>Magn. Field 3</td>
<td>129.0</td>
<td>171.2</td>
<td>119.2</td>
<td>154.4</td>
</tr>
<tr>
<td>Magn. Field 4</td>
<td>132.6</td>
<td>172.2</td>
<td>127.7</td>
<td>143.2</td>
</tr>
<tr>
<td>Control</td>
<td>109.3</td>
<td>168.7</td>
<td>140.1</td>
<td>160.3</td>
</tr>
</tbody>
</table>

The \( W_f \) function based on the maximum eigenvalue \( \lambda_s \) of the matrix type \( H \Gamma^{-1} \) will be used as the test function, \( W_s = \frac{\lambda_s}{1+\lambda_s} \) [16].

When the null hypothesis is true, then the parameters of the distribution of the variable \( W_s \) are \( s, m, \) and \( n \) calculated according to the formulas [15]:

\[
s = \min \left[ r(K), r(M) \right], m = \frac{\left| r(K) - r(M) \right| - 1}{2},
\]

\[
n = \frac{N - r(X) - p - 1}{2}, N = r(a(bc+1)).
\]

For \( s=1 \) random variable \( F = \frac{n+1}{m+1} \frac{W}{1-W} \) has \( F \) distribution with \( v_1 = 2m+2 \) and \( v_2 = 2n+2 \) degrees of freedom. The null hypothesis will be rejected on the level \( \alpha \) of significance if \( W_s > W_{\alpha;m,n} \), where \( W_{\alpha;m,n} \) is above a 100\( \alpha \)-percent critical value from the Pillais tables or from the Hecks monograms.

The hypothesis for the \( Z_T \) contrasts \( H_{Z_T1} : \mu_1 - \mu_0 = 0 \) on the equality of the means of the component models can be treated as a joined comparison of the tested factors with the control levels. In this case, we obtain an answer to the important question about the influence of the experimental factors on a given characteristics. Further inference concerning the comparison of particular means with the
control mean can be carried out using simultaneous multivariate multiple comparisons [16].

NUMERICAL EXAMPLE

The experiment, conducted according to the split-plot scheme was arranged in 3 blocks \((r = 3)\). The varieties of sugar beet \((A, a = 4)\) have been compared, in relation to the seed stimulation with a lamp, laser and an electromagnetic \((B, b = 3)\) field and different doses of this stimulation \((C, c = 4)\). The \(a r\) control plots have been sown with the seeds, which had not been subjected to stimulation. Doses of C irradiation were treated as a factor occurring inside B stimulation, since a lamp impulse is not comparable to a laser impulse or time of the magnetic field influence. Table 1 presents the summarised yields from 3 blocks.

The following hypotheses were tested:
1. \(H_{ZT}\): About the equality of the vectors of the grand means for the control plots and for the plots, where the factors have been used. The null hypothesis assumes, that the tested factors jointly do not influence changes in the values of the considered variables.

2. \(H_A\): About the equality of the vectors of the variety effects.

3. \(H_{B1}\): About the equality of the vectors of the lamp, laser and electromagnetic field effects. The null hypothesis assumes that these three levels of the \(B_1\) factor: lamp, laser and electromagnetic field affect the values of the considered variables in the same way.

4. \(H_C\): About the equality of the vectors of the impulse effects. The null hypothesis assumes that the considered impulse numbers of the factors: lamp, laser and electromagnetic field affect the values of the considered variables in the same way.

5. \(H_{AB1}\): About the equality of the vectors of the interaction effects between variety A and stimulation B. The null hypothesis assumes that these three factors: lamp, laser and electromagnetic field influence the values of the variables of the considered varieties of the sugar beet in the same way.

6. \(H_{ZAB}\): About the equality of the vectors of changes of the effects for the particular varieties caused by the seeds stimulation. The sums of products \((11)\) for the null hypotheses given by matrices \((8)\) and covariance matrices for errors are the following:

\[
G_{E1} = \begin{bmatrix}
123.55 & -54.24 \\
1071.38 & \\
\end{bmatrix}
\]

\[
H_R = \begin{bmatrix}
1101.44 & -55.50 \\
1343.44 & \\
\end{bmatrix}
\]

\[
H_A = \begin{bmatrix}
264887 & 60.40 \\
2020.74 & \\
\end{bmatrix}
\]

\[
G_{E2} = \begin{bmatrix}
588.52 & 36.47 \\
1789.59 & \\
\end{bmatrix}
\]

\[
H_B = \begin{bmatrix}
313.33 & 253.23 \\
240.83 & \\
\end{bmatrix}
\]

\[
H_C = \begin{bmatrix}
155.82 & 103.49 \\
316.49 & \\
\end{bmatrix}
\]

\[
H_{B1} = \begin{bmatrix}
59.00 & 68.79 \\
107.07 & \\
\end{bmatrix}
\]

\[
H_T = \begin{bmatrix}
254.33 & 184.44 \\
133.76 & \\
\end{bmatrix}
\]

\[
H_{AB} = \begin{bmatrix}
854.13 & 462.28 \\
308.41 & \\
\end{bmatrix}
\]

\[
H_{AB1} = \begin{bmatrix}
324.74 & 137.67 \\
104.70 & \\
\end{bmatrix}
\]

\[
H_Z = \begin{bmatrix}
529.40 & 324.61 \\
203.71 & \\
\end{bmatrix}
\]

\[
H_{AC} = \begin{bmatrix}
254.33 & 184.44 \\
133.76 & \\
\end{bmatrix}
\]
The inverses of the covariance matrices for the errors are the following:

\[
G_{E1}^{-1} = \begin{bmatrix}
0.0083 & 0.00042 \\
0.00095 & 0.00083
\end{bmatrix},
\]

\[
G_{E2}^{-1} = \begin{bmatrix}
0.0017 & -0.000035 \\
0.000056 & 0.000042
\end{bmatrix}.
\]

According to the one variable analysis of variance, we will test the hypotheses for each source of variation separately. The maximum eigenvalue \( \lambda_s \) of the matrices \( HG^{-1} \) will be used as the test function.

The hypothesis for the \( Z_T \) contrast was verified by using the \( F \) statistic. For \( s=1 \) random variable \( F = \frac{n+1}{m+1} \frac{W}{1-W} \) has \( F \) distribution with \( v_1 = 2m + 2 = 1 \) and \( v_2 = 2n + 2 = 101 \) d.f.

The \( W_0 \) values exceed the critical \( W_{0.01} \) values (Table 2), which resulted in rejecting the null hypotheses. We have got not an answer to the question which levels of factors and which variables or else which combinations of variables and factors are responsible for rejecting the null hypotheses. A more detailed inference will be possible after using the multiple comparisons. The calculations were made by using a computer Maple program, while a program for the total statistical analysis of the experiments with the control treatment according to the presented theory is under preparation now.

CONCLUSIONS

1. A new form of the model (3) with two sub-models enables the analysis of variance for the tested factors without the control, solely on the basis of the model (2).

2. The analysis of variance for the tested factors, conducted only on the basis of the model (2), gives conclusions comparable with the conclusions from another experiments with the same factors, but without control.

3. Model (3) gives a better visualisation of the experiment than the traditional model.

4. In this new form of the model, control effects are separated in the beginning.

5. In the joined model (3), we have got, \( a r \) degrees of freedom more than in (2) for each variable. Additional degrees of freedom will be used for the comparison of the tested factors with the control, together or separately.

6. The new formula of the model is especially useful, when control treatment means that no treatment is applied to the experimental units, so effects are equal to zero.

REFERENCES