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CRACK TORTUOSITY IN SWELLING CLAY SOILS*

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A b s t r a c t. A relationship between the planar and spatial tortuosities of a statistically isotropic crack network in a clay soil is derived from the fact that connected cracks outline peds and are their boundaries. Connectedness, a parameter characterizing the crack network, determines this relationship. The two- and three-dimensional tortuosities vary from 1.5 to 2.2 and from 1.4 to 3.25, respectively, when connectedness decreases from unity to zero. A method proposed for processing two-dimensional images of crack networks enabled the estimation of two- and three-dimensional tortuosities of an assumed isotropic crack network. Two-dimensional images of seventeen different crack networks, available in the literature, were used to show the application of the proposed dependency between planar and spatial tortuosities.

K e y w o r d s: swelling soil, crack network, crack connectedness, planar tortuosity, spatial tortuosity

INTRODUCTION

One of the characteristics of a crack network in a clay soil is tortuosity which determines the mean length of flow through cracks and influences the hydraulic properties of the soil. Tortuosity definition of crack network can be introduced either as planar tortuosity of a two-dimensional crack network of linear cracks on a soil cross-section or as spatial tortuosity of a three-dimensional crack network of planar cracks. The main purpose of this work is to show that, in the case of a statistically isotropic crack network, there is a relationship between the planar and the spatial tortuosities. It is clear that to estimate experimentally the planar tortuosity from a two-dimensional image of a crack network is much simpler than to estimate the spatial tortuosity from a three-dimensional image. So the relationship may be used for estimation of the spatial tortuosity from the planar one.

First we define the tortuosities of the crack network in the two- and three-dimensional cases. Then we calculate them as functions of each other on the basis of the distribution of ped dimensions and show how to estimate them from an experimental two-dimensional image of a crack network which is assumed to be statistically isotropic. To evaluate the model we used crack network images available in literature.

THEORY

One may consider the crack network in a sufficiently small volume, as well as in any of its cross-sections, as statistically homogeneous. The crack network is characterized by a dimensionless value, c describing the extent of the crack coalescence (the crack network connectedness) and by an average spacing between crack intersections with a straight line, d [1, 2, 4]. The connectedness, c_2 for a cross-sectional plane of the crack network may depend on its

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orientation and differ from the spatial crack network connectedness, c_3 because natural crack networks can be anisotropic [14]. For simplicity, we shall assume that crack networks are locally isotropic when c_2 does not depend on the orientation of the cross-section. In the isotropic and homogeneous case the connectedness c_3 may differ from the connectedness c_2 only because of cracks coplanar with the cross-section under consideration. However, it may be assumed that the number of these is very small compared with the number of cracks intersecting the cross-section, and one may consider $c_2 = c_3 \equiv c$. In the following, results based on the assumption of local crack network isotropy will be compared with experimental data.

In the two dimensional (planar) case, the tortuosity of a crack network, T_2 is defined as:

$$T_2 = L_2 / L_1 \qquad (1)$$

where L_1 is the number of cracks intersecting the side of a unit square and which are connected with other cracks within it forming continuous through (linear) paths to the opposite side (in the following, straight sections of such a path are referred to as (one dimensional) through-connected cracks); L_2 is the total length of through-connected cracks per unit area. Similarly in the three-dimensional case the tortuosity of a (spatial) crack network, T_3 is defined as:

$$T_3 = L_3 / L_2 \qquad (2)$$

where L_2 is the total length of the traces of cracks intersecting the side of a unit cube and connected with cracks within it forming continuous, through (areal) paths to its opposite side (in the following planar sections of such paths are referred to as (two dimensional) through-connected cracks); L_3 is the total surface area of the through-connected cracks per unit volume.

The key point of the approach to estimate the values of L_1, L_2 and L_3 is: connected cracks outline, or nearly outline, peds and are their boundaries. Consequently L_1, L_2, and L_3 may be estimated from the distributions of fragment dimensions in the one-, two- and three-dimensional cases, respectively ("a one dimensional fragment" is the spacing between intersections with a line of two neighboring connected cracks). Using the distributions derived from the model of multiple cracking and fragmentation [1,2,4], with the parameters: connectedness, c and mean crack spacing, d one can get the expression:

$$L_n = B_n(c) / d, \qquad n = 1, 2, 3 \qquad (3)$$

where $B_n(c)$ are dimensionless functions of the connectedness. According to Eqs (1), (2) and (3) the tortuosities are:

$$T_n(c) = B_n(c) / B_{n-1}(c), \quad n = 2, 3. \qquad (4)$$

Equation (4) (for $n = 2$ and 3) shows that the tortuosity T_3, as well as the tortuosity T_2, do not depend on the scale factor (the mean spacing between cracks) d, but only on the connectedness c as the parameter of crack network. It is assumed that the shape of a two- or a three-dimensional fragment is, on the average, a square or a cube, respectively. The line in Fig. 1 shows the dependency $T_3(T_2)$ resulting from the dependencies $T_2(c)$ and $T_3(c)$ for an isotropic and homogeneous crack network. As can be seen, the tortuosity T_3 increases from 1.4 to 3.25 and the tortuosity T_2 from 1.5 to 2.2 when the connectedness c decreases from one to zero.

MATERIALS AND METHODS

In this work we are interested in finding tortuosities T_2 and T_3 from two-dimensional images of crack networks. The images of crack networks can be: a) at the soil surface; b) at different cross-sections of soil cores or blocks; and c) at the surface of specially-prepared soil samples. A number of such images are available in the literature [7,9-14] and were used for evaluation of the model. They are shown in Fig. 2 on a much smaller scale than the originals. They are numbered according to decreasing connectedness. Images 1-4, 6, 9, 11, 13 (from Fig. 2 of Guidi *et al.* [7]) are photographs of eight replicates of artificially-prepared samples of the 1-2 mm fractions of an Italian Fluvisol, >48% clay (images 1-4) and a Regosol, > 46% clay (images 6, 9, 11, 13). Image 5 from Ringrose-Voase and Bullock ([10], Fig. 4(c)) is a photograph of a horizontal cross-section of a Windsor soil, at

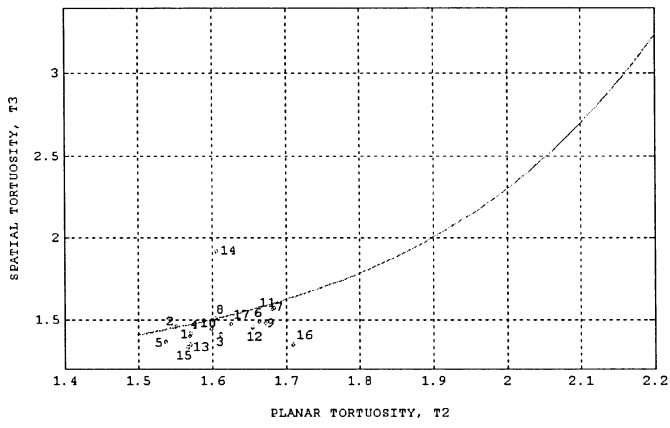

Fig. 1. Predicted mean three-dimensional tortuosity T_3 of an isotropic crack network as a function of the two-dimensional tortuosity T_2 (solid line) and experimental data (points) of clay soils (the numbers refer to soils shown in Fig. 2).

depth 46 cm. Images 7 and 12 are Fig. 7 and the lower part of Fig. 1, respectively, of Scott *et al.* [13]; they are photographs of horizontal cross-sections of clayey subsoils of the Swanwick series at a depth of 46 cm and of the Windsor series at a depth of 35 cm, respectively. Images 8 and 16 are Fig. $1\binom{V}{3}$ and Fig. $1\binom{S}{9}$, respectively, of Ringrose-Voase and Nys [11]; they are photographs of 12.7x10.16 cm horizontal cross-sections of a blocky and a prismatic soil sample, respectively, from North Wales at a depth of 7 cm. Image 10 is Fig. 1 of Ringrose-Voase [9] of a cross-section of a soil from Wales. Images 14 and 15 are two parts of Fig. 1 of Scott *et al.* [12]; the photographs are of 4.8x 7.2 cm soil thin sections cut at random angles of both elevation and azimuth from a profile of the Windsor-series-of-London clay in Essex at depth 35 cm. Image 17 is the right part of Fig. 4(b) of Velde *et al.* [14] which is a photograph of a 5x10 cm soil vertical thin section from a vertisol profile (>65% clay) in Southern Italy at a depth of 155 cm.

The procedure of preparing the images was as follows:

- the original image was magnified to a linear dimension of ~20 cm;
- a strip of the boundary of the magnified image, ~1.5 cm wide, was excluded if it was distorted to allow to get a "working window" square of a side length of 14-23 cm;
- a grid with a spacing of 1cm was drawn on the working window (we designated the total number of the horizontal and vertical lines by i_0).

In the image working window, a line section is considered to be *the trace of a spatial crack* if the ratio of its length to width 3 and the length 2 mm. A straight section is considered to be a *separate* crack. A separate crack whose two ends are connected with other separate cracks is referred to as a *connected* crack, otherwise, as an *isolated* crack. In a series of connected cracks that cross the entire working window, in any direction, each crack is referred to as a *through-conected* crack, otherwise, as a *locally-connected* crack.

The mean two dimensional tortuosity T_2 for the given image is estimated by:

$$T_2 = L_{tc} / L_w N_c \qquad (5)$$

where N_c is the mean number of the intersections of through-connected cracks with a line of length L_w which is the dimension of the window. This mean number was calculated on the basis of i_o profiles of the grid. L_{tc} is the total length of the through-connected cracks, measured within the window.

To obtain data for the estimation of the mean three-dimensional tortuosity T_3, of a statistically homogeneous and isotropic spatial-crack network, one can use any two-dimensional image of this crack network. Considering cracks in a thin layer of thickness h one may

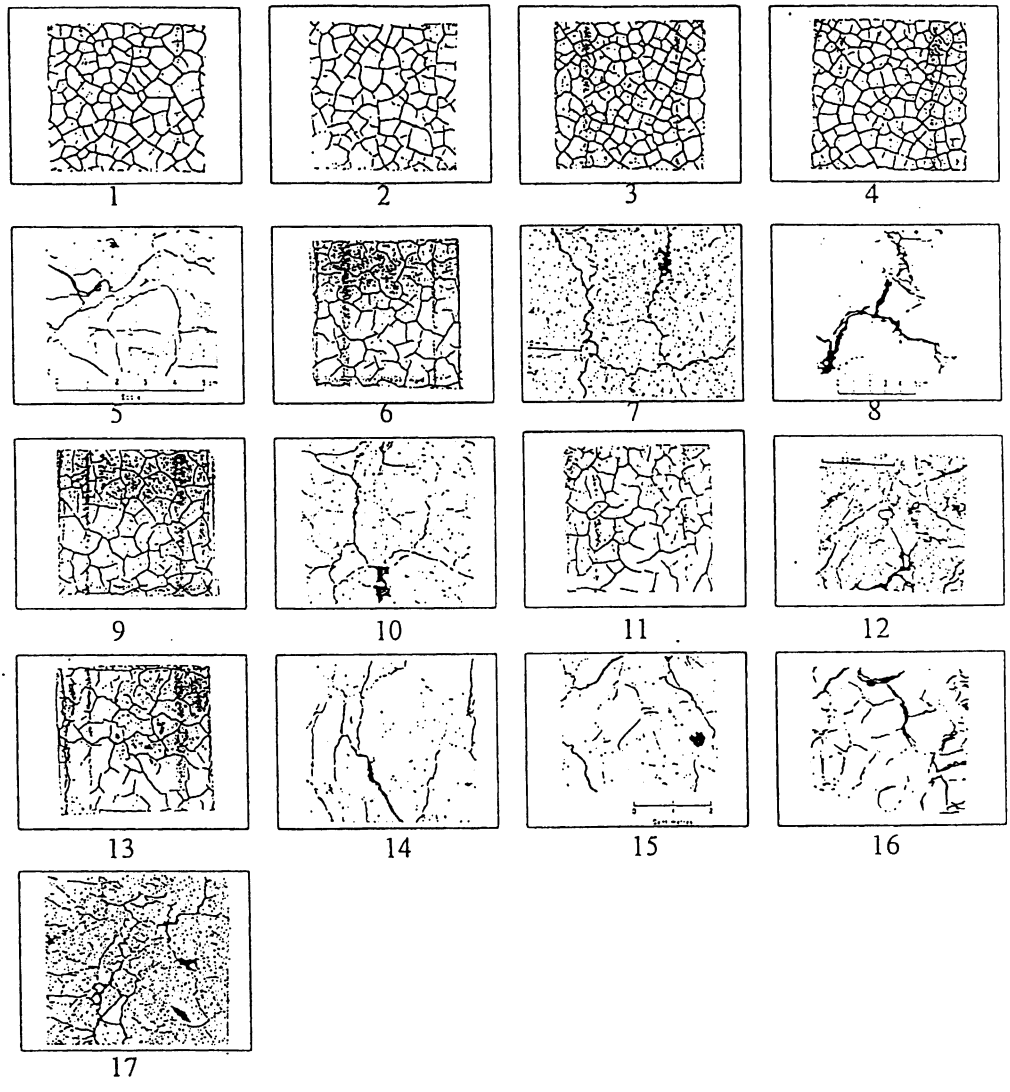

Fig. 2. Two-dimensional crack networks images of clay soils used for experimental estimation of the values T_2 and T_3.

assume that: 1) the spatial crack network in the layer is statistically homogeneous; and 2) each straight section of length l of the two-dimensional image of the crack network on the face of the layer corresponds to a plane crack within it. If the angle between the normal to the layer and the crack plane is θ, then the surface area of the crack in the layer is equal to $lh/\cos\theta$. The total surface area, S_{tc} of all the through, spatially-connected cracks in the layer is:

$$S_{tc} = \sum_i l_i h / \cos\theta_i \qquad (6)$$

where the index i correspond to a separate straight crack trace in the two dimensional image. According to the definition, the tortuosity of the statistically-homogeneous, spatial-crack network is:

$$T_3 = S_{tc} / L_{tc} h = \left(\sum_i l_i / \cos\theta_i \right) / \left(\sum_i l_i \right). \qquad (7)$$

This is the three-dimensional tortuosity in the vicinity of the cross-sectional image under consideration.

For any arbitrary i th connected crack in the layer the corresponding angle θ_i has a random value in the range:

$$0 \leq \theta_i \leq \theta_m \qquad (8)$$

where θ_m is the maximum possible value of the angle θ_i. Herewith $\theta_m < \pi/2$ because the crack must intersect with the cross-section plane. Before estimating the angle θ_m it is worth noting that in the fragment-dimension distribution and the specific fragment surface, any fragment is characterized by its largest dimension x, only. However, the boundary value θ_m of the random values of θ_i is determined by geometrical considerations of the mean fragment shape. To define the three dimensions of a fragment of an arbitrary convex shape we consider the two dimensions of its largest face and the dimension normal to the face and designate them, in the order of decreasing values as x, y, z. The shape of the fragment is characterized by the ratios y/x and z/x. The mean fragment shape, that is the mean shape of a set of fragments (outlined by connected cracks), is characterized by the ensemble averages $\overline{y/x}$ and $\overline{z/x}$ ($\overline{z/x}<\overline{y/x}<1$). Defining $\overline{z/x}\equiv a<1$ then, if the fragment adjoins the surface of the cross-section from below by its (x, y) side (Fig. 3), the angle θ_i, between the normal to the cross-section, n and the fragment face (crack) in the layer, can not surpass arctan (x/z), or otherwise must be, on the average, less than arctan($1/a$). If the adjoining side is (y, z) or (z, x) the limit value of the corresponding angle will be even smaller. Thus:

$$\theta_m = \arctan\,(1/a). \qquad (9)$$

Data from observations by Repin [8], strengthened by theoretical estimates of Chertkov [3] show that for natural rock blocks $\overline{z/x} \equiv a \cong 0.5$. This value may be applied to fragments outlined by shrinkage cracks in drying clay soils as shown by Chertkov [4] (it is noteworthy that for soil aggregates this ratio $\overline{z/x}$ = 0.63-0.65 as

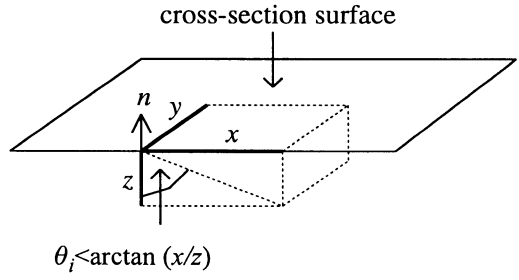

Fig. 3. Sketch for estimating the value of θ_m. Values of $x > y > z$ give the three dimensions of a fragment (ped), of an arbitrary convex shape, adjoining from below to the cross-section surface by the side (x, y). θ_i is the random angle between the normal to the cross-section and the plane of the i th crack.

shown by Dexter *et al.* [6] and Dexter [5]. Because of the experimental character of the value $a \cong 0.5$ one ought to replace a in Eq. (9) by $a - 2\sigma$, where σ is the standard deviation of a (at a 0.95 confidence level). Since the fragment dimensions were measured to an accuracy of 5-7 % that of their ratio was 10-15 %. Hence, $a - 2\sigma \cong 0.35$ and:

$$\theta_m \cong \arctan\,(1/0.35) \cong 70.7^{\circ}. \qquad (10)$$

The procedure for determining the random intersection angle θ_i in Eq. (7) is as follows. As a consequence of the crack-network isotropy, it is assumed that the values of the random angle θ_i for any i are uniformly distributed in the range given by Eq. (8). Moduli of the random azimuths φ_i of the crack traces on the cross-sectional plane are also uniformly distributed in the range $0 \leq |\varphi_i| \leq \pi/2$. So, we may take as a random value of θ_i (if the number of cracks is large):

$$\theta_i = |\varphi_i| \frac{\theta_m}{\pi/2}. \qquad (11)$$

In the numerical estimation of the tortuosity T_3, the angles φ_i were measured relatively to the horizontal side of the working window.

RESULTS AND DISCUSSION

The results of the estimations of T_2 and T_3 for the crack networks of the images in Fig. 2

are shown by the points in Fig. 1 along with the theoretical line. The numbers correspond to number of the image in Fig. 2.

The experimental errors of c, T_2 and T_3 may be estimated as follows. Images 1, 2, 3, and 4 in Fig. 2 are of the same soil and the same conditions [7]. The same is true also for images 6, 9-11, and 13 in Fig. 2 [7] and for images 14 and 15 in Fig. 2 [12]. Hence, it is possible to assess the errors of c, and for each group of images. The standard deviations D_{T_2} , D_{T_3} found for images 1- 4 are: $D_{T_2} \cong 0.03$, $D_{T_3} \cong 0.03$, for images 6, 9, 11, and 13 are: $D_{T_2} \cong 0.05$, $D_{T_3} \cong 0.09$ and for images 14 and 15 are: $D_{T_2} \cong 0.03$, $D_{T_3} \cong 0.04$.

There are three possible sources for the discrepancies between the experimental points and the theoretical curves in Fig. 1:
- the assumption of a square shape for planar fragments and cubic shape for volumetric fragments in the calculations of the mean tortuosities, T_2 and T_3, respectively;
- the assumption of a statistically-isotropic crack network (equal values of connectedness for the two- and three-dimensional cases and calculating T_3 on the basis of the two-dimensional image);
- the procedure for counting the number of cracks of different types within the working window (the ratio of the crack length to width > 3 and the crack length > 2mm).

One should note with regard to the choice of the fragment shapes that when using two alternative simple fragment shapes -triangle and tetrahedron or circle and sphere - the discrepancies between the theoretical curve of T_3 (T_2) and the experimental points in Fig. 1 are much larger and more than three standard deviations. The estimates of the experimental errors of T_2 and T_3 and the distribution of the points around the curves in Fig. 1 show that the discrepancies do not, as a rule, surpass two standard deviations. Hence, one may say that, in spite of the approximations used, results of the suggested model and those of published data of images of crack network in clay soils are in reasonable agreement.

CONCLUSION

The theoretical estimation of tortuosities of planar and spatial crack networks were based on the relation between cracks and fragments - the faces of fragments are defined by connected cracks. With a statistically-isotropic crack network, the two- and three-dimensional tortuosities, T_2 and T_3, respectively depend only on the connectedness c of the crack network and are related to each other. An analytical expression which may be plotted graphically, was found for the function $T_3(T_2)$. A method for processing two-dimensional images of crack networks is proposed for estimating the values of T_2 and T_3 (assuming crack network isotropy). Results obtained from seventeen two-dimensional images published in seven works do not contradict the predicted dependency $T_3(T_2)$.

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